**Supplementary Material**

# Permutation entropy based time series analysis: equalities in the input signal can lead to false conclusions

## Analysis of discretized sequences from continuously distributed pseudorandom numbers

We have also analyzed discretized sequences from uniformly, normally and exponentially distributed pseudorandom numbers. One hundred independent sequences of length $N = 1,000$ have been generated for each one of these continuous distributions with different discretization levels $l$ between 2 and 50. In the discretization process, all values of the time series within a specified range are mapped to a common value. The discretization level $l$ is a parameter that indicates the number of equal-length, contiguous, nonoverlapping segments in which the range of the original time series is divided. Particularly, we define to map all the values that are in a particular range interval to the smallest real value of this interval. Next, a toy example is included to illustrate this procedure. For an arbitrary time series $X_{orig} = \{0.7, 0.5, 0.6, 0.2, 0.1, 0.3, 0.8, 1.0, 0.4, 0.9\}$ and $l = 3$, the discretized time series will be $X_{disc} = \{0.7, 0.4, 0.4, 0.1, 0.1, 0.1, 0.7, 0.7, 0.4, 0.7\}$. Obviously, the number of repeated values in the transformed sequences increases as the number of discretization levels decreases. Results obtained for the normalized permutation entropy $\mathcal{H}_S$ as a function of the number of discretization levels $l$ for the three continuous distributions are depicted below. Mean and standard deviation (displayed as error bars) of the estimated $\mathcal{H}_S$ values with $D \in \{3, 4, 5, 6\}$ and embedding delay $\tau = 1$ for the one hundred realizations are shown. Mean values of the normalized permutation entropy obtained for the one hundred original continuous sequences are also displayed (horizontal dashed lines). It is worth noting here the lower $\mathcal{H}_S$ estimated values for low number of discretization levels when discretized sequences from exponentially distributed pseudorandom numbers are analyzed (please see Fig. 3). This is due to the way the discretization procedure is carried out, *i.e.* more ties are obtained when the original exponentially distributed pseudorandom numbers are transformed. More equalities are also obtained when normally distributed pseudorandom numbers are discretized with $l = 3$ for

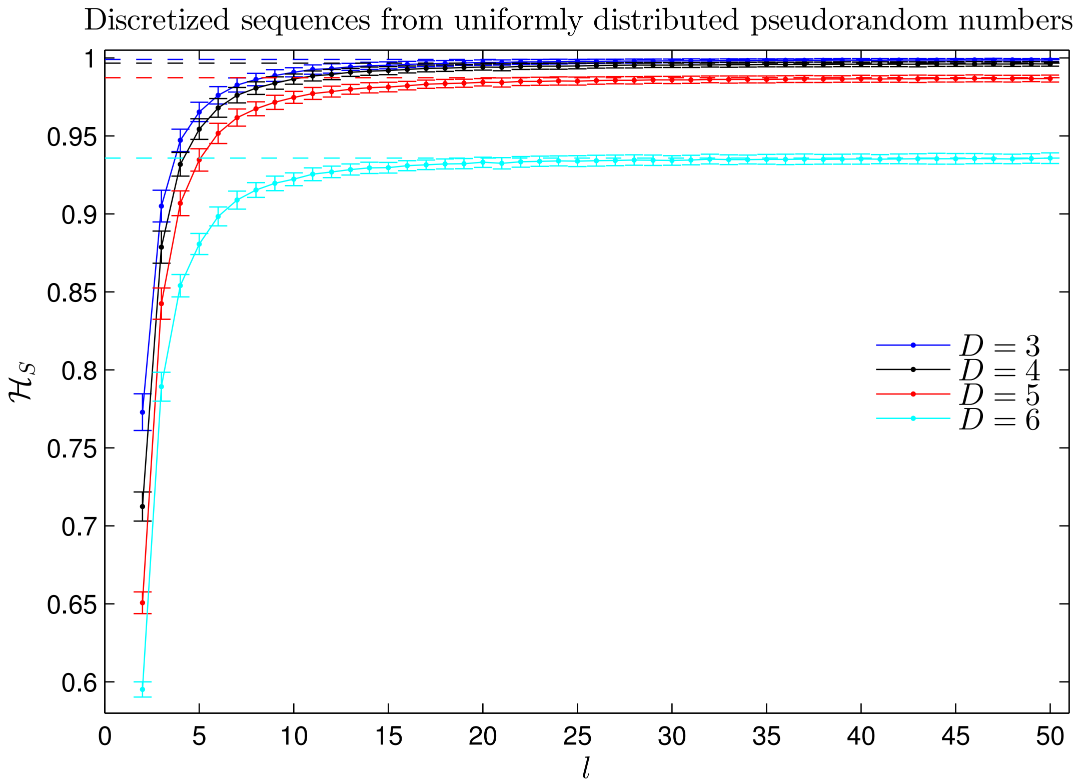


Figure 1: Mean and standard deviation (displayed as error bars) of the normalized permutation entropy $\mathcal{H}_S$ as a function of the number of discretization levels $l$ for one hundred independent discretized sequences from uniformly distributed pseudorandom numbers of length $N = 1,000$. Results obtained for different embedding dimensions ($D \in \{3, 4, 5, 6\}$) and embedding delay $\tau = 1$ are included. Horizontal dashed lines indicate the mean value of $\mathcal{H}_S$ estimated values for the original continuous sequences.

$D = 3$ (please see Fig. 2).

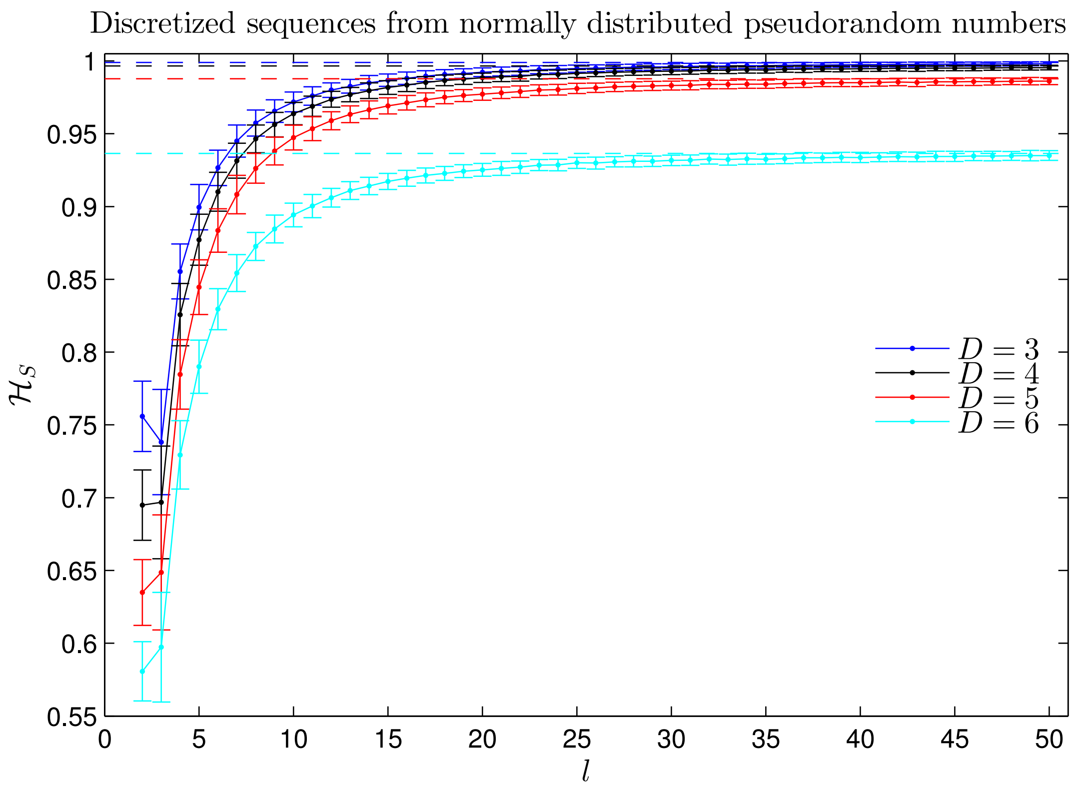


Figure 2: Same as Fig. 1 but for discretized sequences from normally distributed pseudorandom numbers.

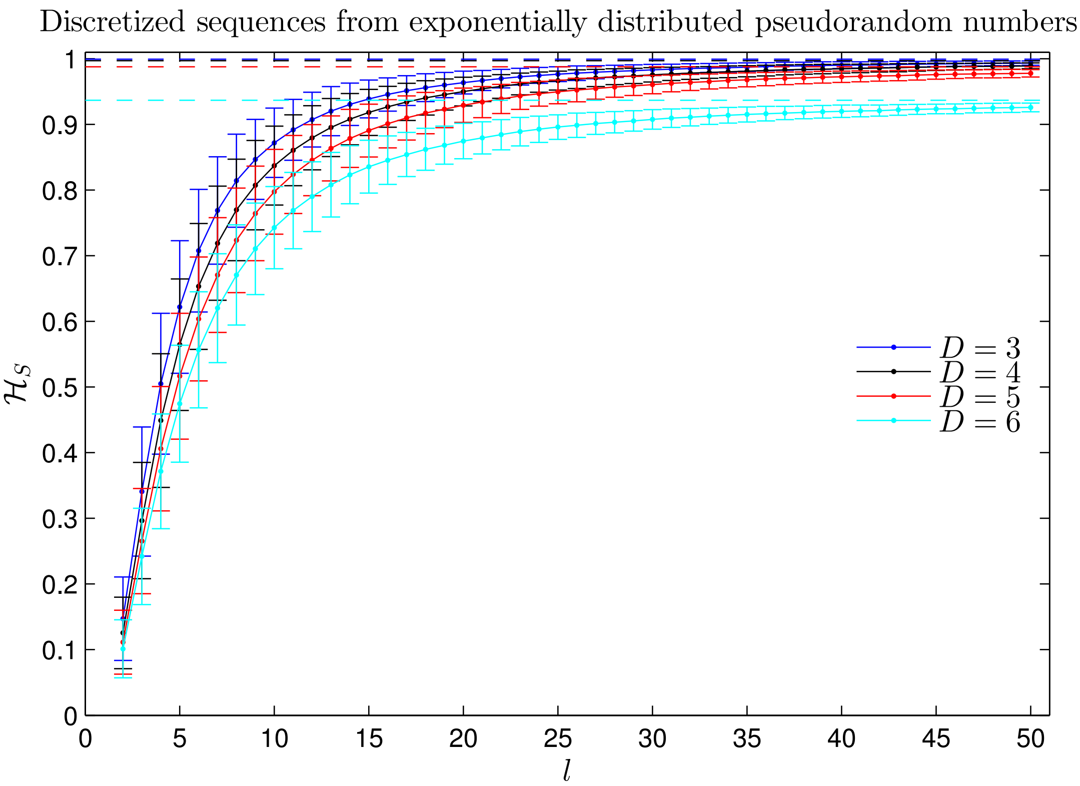


Figure 3: Same as Fig. 1 but for discretized sequences from exponentially distributed pseudorandom numbers.