

# Tilt dynamics of an electrostatically actuated microoscillator at a liquid-liquid interface

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**Abstract.** We investigate the time-domain tilt response of an electrostatically actuated mechanical microoscillator positioned at a liquid-liquid interface. An analytical model is presented to simulate the microoscillator's rotational motion inside a microchannel completely filled with two immiscible liquids. The model considers two coupled ordinary differential equations; one simulates the mechanical response of the microplate-microbeam assembly making-up the microoscillator and the other provides the behaviour of the electrical charge responsible for the electrostatic moment that tilts the microplate. Results show that remarkable improvements in sampling time and sensitivity can be obtained using a bi-liquid configuration versus its single-liquid counterpart. Therefore, enhanced performance of mechanical microsensors for liquids could be achieved.

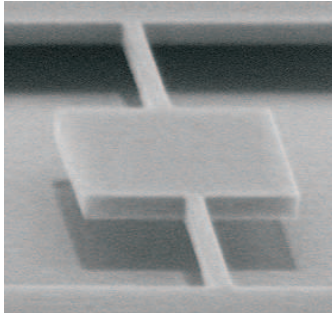
*Keywords:* microfluidics, surface tension, liquid interface, microoscillator, MEMS.

## 1. Introduction

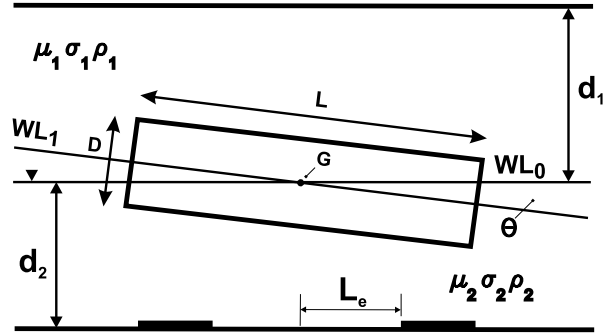
Mechanical microoscillators are used extensively as components in MEMS (Micro Electro Mechanical Systems) that operate with liquids and gases. Their precision, inexpensive fabrication and minute fluid sample requirements are some of the features that generated a considerable volume of research in engineering and sciences. When operating with liquids, it was recognized [1] that resonant frequencies and quality factors  $Q$  of microoscillators were very small compared with those in air due to higher viscous effects. As a consequence, rapidly decaying dynamic responses lead to short sampling times and low sensitivity to determine, for instance, a resonant frequency shift due to adsorption of a target substance.

To circumvent these deficiencies, a considerable number of works was carried out. To name just a few contributions, some investigators used feedback forces applied through the gain of a feedback circuit [2, 3] to change the effective damping of the dynamic system. Alternatively, some works used an applied voltage to produce a dynamic change of the spring constant of microcantilevers [4] and of torsional microresonators [5]. In an entirely different approach, liquid was confined to microchannels inside oscillating microbeams operating in air [6], thus avoiding operation in a liquid medium.

In this work, we further our investigation of a simpler and novel concept [7, 8, 9] where surface tension is positively used to improve performance of mechanical microoscillators. In this work, we present an analytical solution to the Lagrangian equations resulting from the application of Hamilton's principle to the proposed electromechanical system. The study of the tilt dynamics



**Figure 1.** SEM of microoscillator in air ( $L = 2\mu m$ ). Reprinted with permission from [5], Copyright 2000, Amer. Inst. of Physics.



**Figure 2.** Side view of rotated microplate at liquid-liquid interface showing position of electrodes (microbeam not shown).

of a torsional microbridge demonstrates that improvements by several orders of magnitude can be achieved for extremely small resonators operating at the interface between two immiscible liquids.

## 2. Statement of the Problem

Figure 1 shows a Scanning Electron Micrograph (SEM) of a torsional microresonator in air. In our case, Figure 2, this microplate-microbeam assembly is located at the interface of two immiscible liquids ( $WL_0$ ). Immediately below the microplate, two electrodes are located at a distance  $L_e$  from a torsional axis co-axial with the microbeams' axis that passes through the center of gravity  $G$ . The liquid-liquid interface is coplanar with the microplate's horizontal mid-plane. Upper and lower liquids have viscosity, surface tension and density triplets  $(\mu_i, \sigma_i, \rho_i)$ , with  $i = 1, 2$ . Application of a voltage  $V_e$  between the assembly and one electrode produces an accumulation of charge  $q(t)$  in a manner equivalent to that of a parallel moving plate capacitor [10]. The resulting electrostatic moment causes rotation of the microoscillator to a maximum value  $\theta(t = t_a) = \theta_a$  (position  $WL_1$  in Figure 2). When  $V_e$  is removed, the microoscillator returns to its original initial position at rest. We limit the value of  $V_e$  to 1.5V so as to minimize the possibility of lower liquid ionization.

Application of Hamilton's principle to the resulting electromechanical system yields both Lagrangian equations for the computation of  $q(t)$  and  $\theta(t)$ . For the accumulation of charge  $q(t)$  the equation reads

$$L_i \ddot{q} + R \dot{q} + \frac{q}{C} = V_e, \quad (1)$$

$$C = \frac{\epsilon A}{d_2 - \frac{D}{2} - (\frac{L}{4} + \frac{L_e}{2}) \tan \theta}, \quad (2)$$

where  $L_i$  and  $R$  are the circuit's inductance and resistance respectively, and  $\epsilon$  is the permittivity of the lower liquid.  $L$  and  $D$  are the microplate's side length and thickness respectively,  $d_2$  is the depth of the lower liquid and  $A = (L/2 - L_e) \times B$ . The underlying assumption in (2) is that the electric field is uniform across the gap between the microplate and the electrode. Field lines are assumed perfectly vertical generating equipotential lines parallel to the microplate/electrode planes. In practice, such parallelism is distorted at the ends of the plates due to non-uniformity

of the electric field. However, such distortion can be readily neglected if the separation gap is small compared to the length  $L/2 - L_e$ .

For the mechanical microoscillator, the equation that models its tilt response reads:

$$a_1 \ddot{\theta} + a_2 \dot{\theta} + a_3 \theta = \frac{(\frac{L}{4} + \frac{L_e}{2}) q^2}{2 \epsilon A \cos^2 \theta}. \quad (3)$$

The coefficient  $a_1$  is composed of three terms due to the plate's moment of inertia, the added inertia from surface tension and the fluids' added mass, and can be obtained as [9]:

$$\begin{aligned} a_1 = & \rho^p L B D \frac{(D^2 + L^2)}{12} + \\ & + 2(\sigma_2 - \sigma_1)(L + B) \frac{(D^2 + L^2)}{12g} + \\ & + \frac{\pi}{256}(\rho_1 + \rho_2)L^4 B, \end{aligned} \quad (4)$$

where  $B$  is the dimension of the microplate in the direction across the microchannel,  $\rho^p$  is the density of the microbeams/microplate material (Pt in this work) and  $g$  is the acceleration of gravity.

Contributions to squeezed film damping are from the fluid regions between the microplate and the substrates immediately above and below. They can be computed as [11]:

$$\begin{aligned} a_2 = & \left\{ \left[ \frac{B[L + 1.65(d_1 - T_1)]^4 \mu_1}{15L(d_1 - T_1)^3} + 3.2\mu_1 B \sqrt{\frac{L + 2.7D}{d_1 - T_1}} \right] + \right. \\ & \left. + \left[ \frac{B[L + 1.65(d_2 - T_2)]^4 \mu_2}{15L(d_2 - T_2)^3} + 3.2\mu_2 B \sqrt{\frac{L + 2.7D}{d_2 - T_2}} \right] \right\} \left\{ \frac{L}{2} \right\}^2, \end{aligned} \quad (5)$$

where  $T_1, T_2$  are the vertical distances from the liquid-liquid interface to the upper and lower surfaces of the microplate respectively.

Finally, the contributions to the restoring term are from buoyancy, torsional moments of the microbeams and surface tension respectively. Their final form is [9]:

$$\begin{aligned} a_3 = & \frac{gLB}{12} \left\{ \rho_2[L^2 + 6T_2(T_2 - D)] - \rho_1[L^2 + 6T_1(T_1 - D)] \right\} + \\ & + 2 \frac{\beta l_b t_b^3 G}{b_b} + \\ & + 2L \left\{ \sigma_2 \frac{L^2 + 6T_2(T_2 - D)}{12T_2} - \sigma_1 \frac{L^2 + 6T_1(T_1 - D)}{12T_1} \right\} \left[ \frac{1 - \cos\theta(t)}{\cos\theta(t)} \right], \end{aligned} \quad (6)$$

where  $l_b, t_b$  are the cross-sectional dimensions of the microbeams,  $b_b$  is their length,  $G$  is their shear modulus, and  $\beta$  is a coefficient of proportionality that depends on the ratio  $l_b/t_b$ [12].

### 3. Proposed solution

In this section, we present the analytical solutions for equations (1) and (3) when the microplate is electrostatically actuated by a constant voltage  $V_e$ . For the oscillators' sizes considered in this work, angles of rotation are sufficiently small to warrant the assumptions  $\tan\theta \approx 0$  in (1)

and  $\cos^2\theta \approx 1$  in (3). Noting that  $(\frac{a_3}{a_1}) - (\frac{a_2}{2a_1})^2 > 0$  must hold for the assembly to oscillate, the solutions of (1) and (3) when the capacitor is charging can be obtained using homogeneous initial conditions:

$$q^c(t=0) = 0, \quad \dot{q}^c(t=0) = 0, \tag{7}$$

$$\theta^c(t=0) = 0, \quad \dot{\theta}^c(t=0) = 0, \tag{8}$$

as:

$$q^c(t) = c_1^{qc} e^{\lambda_1^{qc} t} + c_2^{qc} e^{\lambda_2^{qc} t} + c_3^{qc}, \tag{9}$$

$$\begin{aligned} \theta^c(t) = & e^{-\frac{a_2}{2a_1} t} \left\{ D_1^{\theta c} \sin \left[ \sqrt{\frac{a_3}{a_1} - \left(\frac{a_2}{2a_1}\right)^2} t \right] + D_2^{\theta c} \cos \left[ \sqrt{\frac{a_3}{a_1} - \left(\frac{a_2}{2a_1}\right)^2} t \right] \right\} + \\ & + c_0^{\theta c} + c_1^{\theta c} e^{\lambda_1^{\theta c} t} + c_2^{\theta c} e^{\lambda_2^{\theta c} t} + c_3^{\theta c} e^{\lambda_3^{\theta c} t} + c_4^{\theta c} e^{\lambda_4^{\theta c} t} + c_5^{\theta c} e^{\lambda_5^{\theta c} t}. \end{aligned} \tag{10}$$

At certain  $t = t_a$ , the capacitor reaches a steady state charge  $q_a$ . Subsequently, at  $t = t_b$  the microplate stops moving at  $\theta = \theta_a$ . At this point, the voltage  $V_e$  is removed and the capacitor discharges. The initial conditions to solve (1) and (3) are then:

$$q^d(t = t_b) = q_a, \quad \dot{q}^d(t = t_b) = 0, \tag{11}$$

$$\theta^d(t = t_b) = \theta_a, \quad \dot{\theta}^d(t = t_b) = 0, \tag{12}$$

and the solutions become:

$$q^d(t) = c_1^{qd} e^{\lambda_1^{qd} t} + c_2^{qd} e^{\lambda_2^{qd} t}, \tag{13}$$

$$\begin{aligned} \theta^d(t) = & e^{-\frac{a_2}{2a_1} t} \left\{ D_1^{\theta d} \sin \left[ \sqrt{\frac{a_3}{a_1} - \left(\frac{a_2}{2a_1}\right)^2} t \right] + D_2^{\theta d} \cos \left[ \sqrt{\frac{a_3}{a_1} - \left(\frac{a_2}{2a_1}\right)^2} t \right] \right\} + \\ & + c_1^{\theta d} e^{\lambda_1^{\theta d} t} + c_2^{\theta d} e^{\lambda_2^{\theta d} t} + c_3^{\theta d} e^{\lambda_3^{\theta d} t} \end{aligned} \tag{14}$$

Expressions for all constants in (9), (10), (13) and (14) are provided in the Appendix.

#### 4. Results

In this section, we present results for charge  $q$  and tilt  $\theta$  as a function of time. Given the large number of parameters involved in the problem at hand, we restrict ourselves to varying only the data provided in Tables 1 and 2. Where appropriate, we have taken the quality factor to be  $Q = a_3/(2\pi f_0 a_2) = \sqrt{a_3 a_1}/a_2$ , and the undamped natural frequency of the oscillator as  $f_0 = [1/2\pi] \sqrt{a_3/a_1}$ .

Figure 3 shows the electromechanical response of the assembly at a water-silicon oil interface (W-SO1) and in water only (W-W) for  $L = 1\mu m$ . The coefficients in (1) are such that cause the charge  $q$  to grow exponentially and very rapidly, with full charge condition reached in around

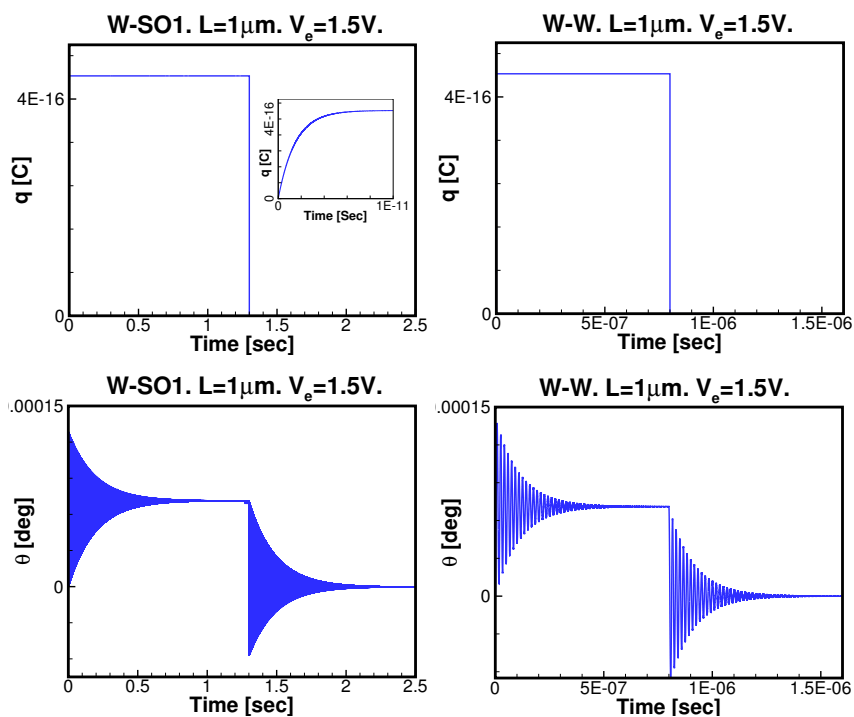
**Table 1.** Problem data (all lengths in  $\mu m$ ), material Pt.

Microbeam			Microplate			Microchannel		Electrical			
$l_b$	$b_b$	$t_b$	$L$	$B$	$D$	$d_1$	$d_2$	$L_e$	$L_i$ [H]	$R$ [ $\Omega$ ]	$V_e$ [V]
0.2	$L/2$	0.2	As in Figs.	$L$	0.2	As in Figs.	$d_1$	$L/3$	$1.88 \times 10^{-14}$	5000	1.5

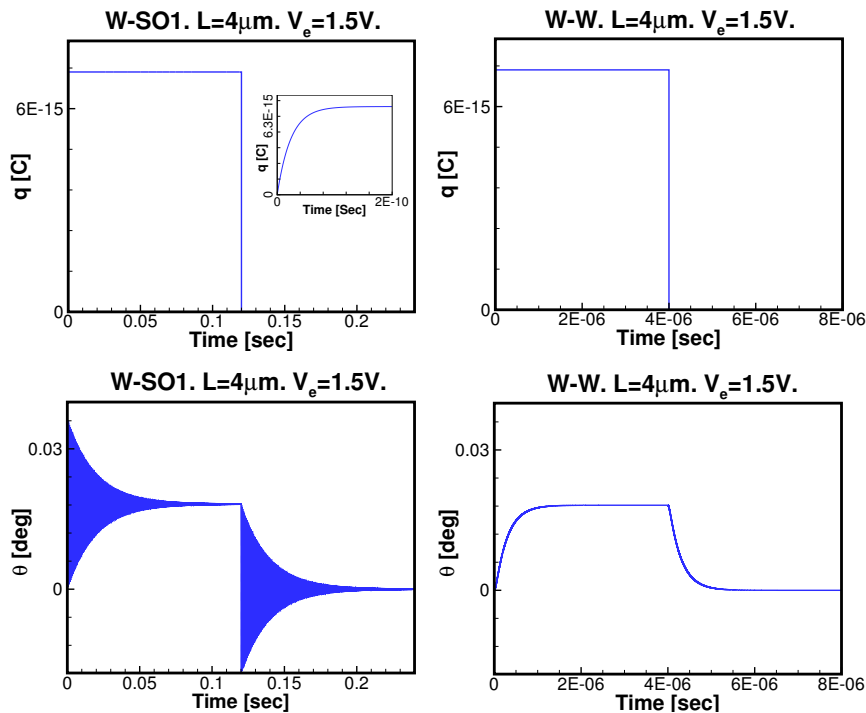
**Table 2.** Liquid properties.

Case	Liquid	Density $\rho$ [ $kg\ m^{-3}$ ]	Kin. Viscosity $\nu$ [ $m^2\ sec^{-1}$ ]	Dyn. Viscosity $\mu$ [ $Nm^{-2}\ sec$ ]	Surface Tension $\sigma$ [ $Nm^{-1}$ ]
SO1	Silicon Oil 1	$0.91 \times 10^3$	$5.0 \times 10^{-6}$	$4.55 \times 10^{-3}$	$19.7 \times 10^{-3}$
OO	Olive Oil	$0.92 \times 10^3$	$91.3 \times 10^{-6}$	$84.0 \times 10^{-3}$	$32.0 \times 10^{-3}$
W	Water	$1.0 \times 10^3$	$1.0 \times 10^{-6}$	$1.0 \times 10^{-3}$	$72.8 \times 10^{-3}$

8 picosec (shown in the Figure's inset). The resulting impulsive electrostatic moment in (3) causes the tilt response to overshoot to a maximum value before its oscillatory decay to a constant  $\theta_a \approx 7 \times 10^{-5}$  for both W-SO1 ( $f_0 = 2.744 \times 10^4 Hz$ ,  $Q = 1.644 \times 10^4$ ) and W-W ( $f_0 = 5.971 \times 10^7 Hz$ ,  $Q = 20.961$ ). For W-SO1, the amount of time required for this periodic



**Figure 3.** Electromechanical response for water-silicon oil interface (W-SO1) and water only (W-W).  $L = 1 \times 10^{-6}m$ ,  $d_1 = 0.5 \times 10^{-6}m$ .



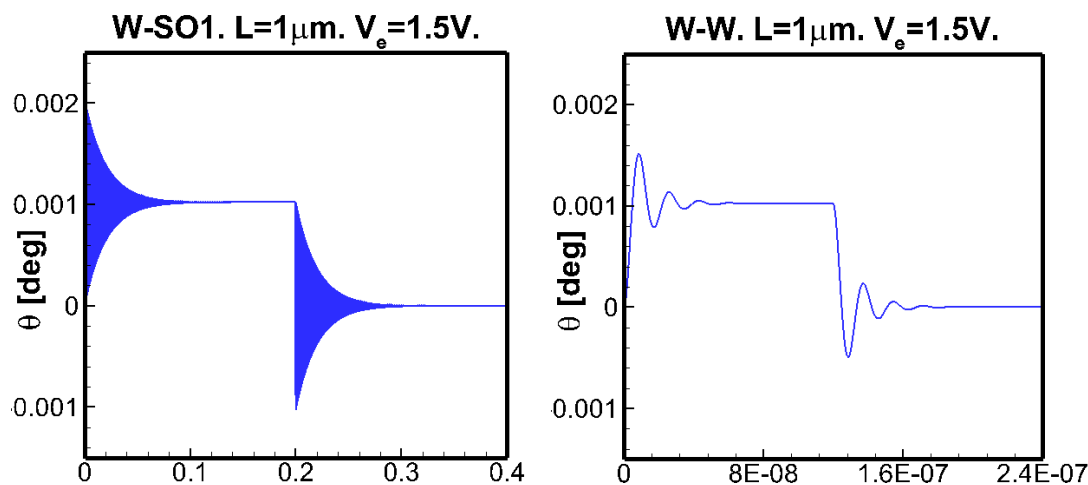
**Figure 4.** Electromechanical response for water-silicon oil interface (W-SO1) and water only (W-W).  $L = 4 \times 10^{-6}m$ ,  $d_1 = 0.5 \times 10^{-6}m$ .

decay to take place is  $t_a \approx 1$  sec. However, for W-W is  $t_a \approx 7.25 \times 10^{-7}$ , a remarkable difference of six orders of magnitude caused by the effect of surface tension. The obvious consequence for a microdevice working with liquid-liquid interfaces is a much greater sampling time as the assembly oscillates, together with a near one thousand-fold increase in the quality factor  $Q$ . We note that, when the voltage  $V_e$  is removed, the charge  $q$  also decays very rapidly, causing the assembly to oscillate back to equilibrium initial conditions  $\theta = \dot{\theta} = 0$ . This second periodic motion due to the capacitor's discharge should be used as a verification of, for instance, the resonant frequency shift due to adsorption of a target substance.

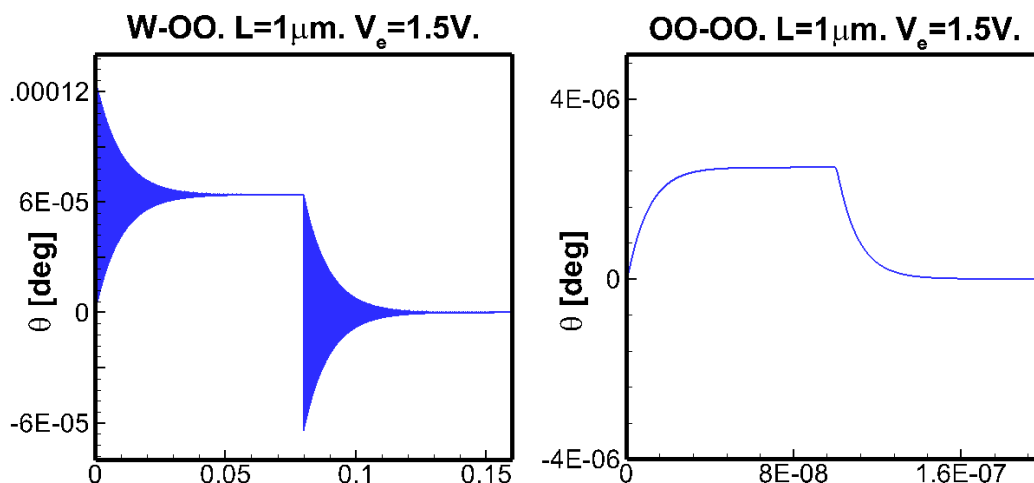
Figure 4 shows electromechanical response of the assembly at a water-silicon oil interface (W-SO1) ( $f_0 = 1.747 \times 10^3 Hz$ ,  $Q = 1.071 \times 10^2$ ) and in water only (W-W) for  $L = 4\mu m$ . For this assembly of larger dimensions, the mechanical response in a single liquid is no longer oscillatory, confirming previous evidence of mechanical microoscillator's poor performance in liquids. In practice, this behaviour would render ineffective a microdevice working on the principle of resonant frequency shift or on dynamic response. On the other hand, a microdevice based on a bi-liquid concept would still serve its intended purpose.

To ease microfabrication, it is sometimes convenient to decrease the depth-width ratio of a microchannel. Figure 5 shows the effect on the response of reduced depth in the microchannel for the microresonator of Figure 3. It can be seen that the consequences of microplate-electrode gap reduction from  $4\mu m$  to  $1\mu m$  are a significant increase in damping and a strikingly diminished time  $t_b$  to reach steady state conditions. For (W-SO1) ( $f_0 = 2.744 \times 10^4 Hz$ ,  $Q = 1.723 \times 10^3$ ) there is a one order of magnitude reduction in the quality factor. However, the microresonator still fulfills its intended purpose. However, for a single liquid (W-W) the microdevice is no longer useful in a dynamic operative mode.

Figure 6 shows results for a different liquid-liquid combination: water and olive oil ( $f_0 =$



**Figure 5.** Electromechanical response for water-silicon oil interface and water only (W-W).  $L = 1 \times 10^{-6}m, d_1 = 0.2 \times 10^{-6}m$ .



**Figure 6.** Electromechanical response for water-olive oil interface and olive oil only (OO-OO).  $L = 1 \times 10^{-6}m, d_1 = 0.2 \times 10^{-6}m$ .

$3.130 \times 10^4 Hz, Q = 940.8$ ). The later fluid exhibits an over 20-fold increase in viscosity and approximately 50% greater surface tension. These property changes cause much greater viscous effects and a considerably decreased time  $t_b$  to reach steady state conditions, as well as an increase in the microoscillator's natural frequency with respect to the case of Figure 3. When the single liquid is olive oil alone (OO-OO) the assembly microplate-microbeam no longer oscillates, again rendering the device totally ineffective.

As a final remark, we note that surface properties, most notably wetting, have considerable effects on the motion and control of liquids inside micrometer scale channels. Each substrate of our microdevice is intended to be made of hydrophilic glass. Despite much controversy in the

literature, there are indications that platinum, the material used for the microbeams and the microplate, has a hydrophilic nature with near zero contact angle in the absence of inorganic and organic contaminants [13]. Contamination caused by some of the liquids used in this work might induce hydrophobicity in platinum, leading to an increase of the contact angle. In such situation, the zero-contact angle assumption in the surface tension terms implied in (4) and (6) should be revised. Higher contact angles would result in somewhat reduced surface tension forces, which will affect natural frequencies and quality factors  $Q$  accordingly.

## 5. Conclusions

An analytical model was developed to study time-domain tilt response of an electrostatically actuated torsional microoscillator positioned at a liquid-liquid interface. Two coupled ordinary differential equations were developed to simulate the tilt response of the microoscillator and the electrical charge of the electromechanical system. Viscosity, density and surface tension of both liquids were taken into account. Several orders of magnitude improvements in sampling times and sensitivity were observed with respect to a single liquid configuration. As a result, much improved sensors based on dynamic response or on resonant frequency shifts could be developed using the concept proposed in this work.

## Acknowledgments

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## Appendix

### Capacitor charging

$$\begin{aligned}
 c_1^{qc} &= -(c_2^{qc} + V_e C), \quad c_2^{qc} = \frac{V_e C \lambda_1^{qc}}{\lambda_2^{qc} - \lambda_1^{qc}}, \quad c_3^{qc} = V_e C \\
 \lambda_1^{qc} &= -\frac{R}{2L_i} + \sqrt{\left(\frac{R}{2L_i}\right)^2 - \frac{1}{CL_i}}, \quad \lambda_2^{qc} = -\frac{R}{2L_i} - \sqrt{\left(\frac{R}{2L_i}\right)^2 - \frac{1}{CL_i}} \\
 \lambda_1^{\theta c} &= \lambda_1^{qc}, \quad \lambda_2^{\theta c} = \lambda_2^{qc}, \quad \lambda_3^{\theta c} = \lambda_1^{qc} + \lambda_2^{qc}, \quad \lambda_4^{\theta c} = 2\lambda_1^{qc}, \quad \lambda_5^{\theta c} = 2\lambda_2^{qc} \\
 c_0^{\theta c} &= \frac{B_0^c a_1}{a_3}, \quad c_i^{\theta c} = \frac{B_i^c}{(\lambda_i^{\theta c})^2 + \frac{a_2}{a_1} \lambda_i^{\theta c} + \frac{a_3}{a_1}} \\
 D_1^{\theta c} &= \frac{1}{\sqrt{\frac{a_3}{a_1} - \left(\frac{a_2}{2a_1}\right)^2}} \left( \frac{a_2 D_2^{\theta c}}{2a_1} - \sum_{i=1}^5 c_i^{\theta c} \lambda_i^{\theta c} \right) \\
 D_2^{\theta c} &= -\left( c_0^{\theta c} + \sum_{i=1}^5 c_i^{\theta c} \right) \\
 B_0^c &= (c_3^{qc})^2 \beta, \quad B_1^c = 2c_3^{qc} c_1^{qc} \beta, \quad B_2^c = 2c_3^{qc} c_2^{qc} \beta, \quad B_3^c = 2c_1^{qc} c_2^{qc} \beta, \quad B_4^c = (c_1^{qc})^2 \beta, \quad B_5^c = (c_2^{qc})^2 \beta \\
 \beta &= \frac{\frac{L}{2} + L_e}{4\epsilon A a_1}
 \end{aligned}$$

### Capacitor discharging

$$\lambda_1^{qd} = \lambda_1^{qc}, \quad \lambda_2^{qd} = \lambda_2^{qc}$$



$$\begin{aligned}
 c_1^{qd} &= -\frac{c_2^{qd} \lambda_2^{qd}}{\lambda_1^{qd}}, \quad c_2^{qd} = \frac{q_0 \lambda_1^{qd}}{\lambda_1^{qd} - \lambda_2^{qd}} \\
 \lambda_1^{\theta d} &= 2\lambda_1^{qd}, \quad \lambda_2^{\theta d} = 2\lambda_2^{qd}, \quad \lambda_3^{\theta d} = \lambda_1^{qd} + \lambda_2^{qd} \\
 c_i^{\theta d} &= \frac{B_i^d}{(\lambda_i^{\theta d})^2 + \frac{a_2}{a_1} \lambda_i^{\theta d} + \frac{a_3}{a_1}} \\
 D_1^{\theta d} &= \frac{1}{\sqrt{(\frac{a_3}{a_1}) - (\frac{a_2}{2a_1})^2}} \left( \frac{a_2 D_2^{\theta d}}{2a_1} - \sum_{i=1}^5 c_i^{\theta d} \lambda_i^{\theta d} \right), \quad D_2^{\theta d} = \theta_0 - \sum_{i=1}^3 c_i^{\theta d} \\
 B_1^d &= (c_1^{qd})^2 \beta, \quad B_2^d = (c_2^{qd})^2 \beta, \quad B_3^d = 2c_1^{qd} c_2^{qd} \beta
 \end{aligned}$$

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