FAULT-TOLERANT SYSTEMS FOR UNMANNED MULTIROTOR AERIAL VEHICLES

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Abstract

This chapter presents some recent results on fault-tolerant control systems for unmanned aerial systems, in particular for multirotor-type vehicles, commonly known as drones. Over the last years, these vehicles have become widely popular. Simplicity and cost-effectiveness have turned out to be very appealing and, as a consequence, an increasing number of applications have risen in many fields such as agriculture, surveillance, and photography, among others. As mission requirements become more demanding, the matter of fault tolerance emerges as a key challenge, especially if system certification is sought.

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Here, the focus is placed particularly on rotor failures in multirotor vehicles, and a specific definition for fault tolerance is considered based on the maneuverability capabilities in case of a failure. A geometric analysis is presented to evaluate the fault tolerant capabilities of a given vehicle, together with an experimental validation. Then, the limitations of this concept are analyzed. Finally, a novel reconfigurable structure is proposed for a fault-tolerant hexarotor, that presents good flight performance in failure cases, together with experimental results.

Keywords: Fault tolerance, fault detection and isolation, multirotor aerial vehicles, unmanned aerial systems

1. INTRODUCTION

Since the early 2000s, when the first commercial radio-controlled small quadrotor entered the market (the Draganflyer quadcopter in 1999, despite there was also a less-known small japanese quadcopter in 1991, the Keyence Gyrosaucer II E-570), the popularity and availability of multirotor aerial vehicles have grown exponentially. At first mainly used by hobbyists, driven by curiosity over the new technology and willing to test its limits, this novelty soon proved to present practical advantages over other flying systems. For example, the vertical take-off and landing (VTOL) capability of these vehicles, allowed them to be easily operated indoors or in reduced spaces. On the other hand, the ability to hover at any point during flight allows for a higher degree of safety for inexperienced users, as the aircraft can remain almost motionless with no commanded actions.

Around 2006, the Federal Aviation Administration (FAA) of the U.S.A. issued the first authorization to use unmanned aerial systems (UAS) and unmanned aerial vehicles (UAV) for commercial applications, and manufacturers of multirotors started to develop specific commercial products aimed at different sectors, such as the movie industry, where the use of these vehicles began to replace manned helicopters, or inspection of difficult-access civil structures (bridges, power lines, buildings) [1]. This also encouraged the production of multirotor vehicles for civil applications, mainly used for personal entertainment, but with an increasing focus on the field of aerial photography and filming. Several companies started designing ready-to-fly products that came with the full package: optimized multirotor design (mainly quadrotors, but also hexarotors), flight controllers with inertial sensors, digital compass and GPS, remote controller and, frequently, first-person view (FPV) video systems. These systems provided a friendly interface to configure the vehicle, as well as a step-by-step set of simple calibration instructions, which allowed even inexperienced users to have a fully functional system in just a few hours, and a professional user to have it ready in a matter of minutes. Also, different companies started producing autopilot controllers, small boards that contained a microcontroller and several sensors that could be configured to control different unmanned systems, were they aerial, terrestrial or aquatic. For UAS, features could range from a simple remote-controlled manual flight to fully autonomous solutions where a preset path could be given for the vehicle to follow. As the use of these products became massive, concerns about individuals' safety and collateral damage in case of accidents began to arise.

Before 2013, most of the (reported) drone related accidents in the U.S. territory were caused by military experimental drones in isolated areas near military bases. At the beginning of that year, a few companies released to the market quadrotor vehicles aimed at amateurs and hobbyists, that allowed to mount small high-definition digital video recorders (e.g. a GoPro camera) for aerial film-making and photography. At a reasonable cost, these products were quite successful, as they were also quickly adopted for professional uses. During that year, the number of reported drone accidents related to civilian-owned systems went up and started to be comparable to the number of military drones accidents, only to exceed them a couple of years later.

These accidents called attention on the need for stricter regulations for UAS that flew over crowds or near restricted zones. In 2014, some of the commercial UAV flight controllers manufacturers added a *no-fly* feature, that prevented the vehicle from entering pre-established restricted zones, even if the pilot wanted to fly manually into them.

It was only recently, in October 2017, when the first waiver to fly a UAV over people was granted by the FAA to the news network CNN [2] to be used for news coverage. The aircraft was the Snap, from Vantage Robotics, a lightweight vehicle, with shrouded blades, held together using magnets which, on impact, should come apart and minimize possible damages. While this is not a fault tolerant system but rather a safety measure to prevent harm to third parties, it presents a practical solution to one of the main dangers of flying in public spaces.

Currently, the manufacturer DJI is in the process of implementing ADS-B receivers on all its UAS above 250g, that will allow to detect nearby aircraft, thus creating dynamically no-fly zones and therefore increasing airspace safety [3].

There exist several lines of work regarding fault tolerance in multirotors that deal with many different failures in this type of systems. However, there are no clear requirements in any country regarding this issue, neither for personal or professional use of such vehicles.

The next section introduces the working principles of multi-rotors necessary to develop the main ideas of the chapter.

1.1. Working Principles of Multirotors

Generally, a multirotor is an aircraft with three or more rotors, where the flight control is based on the speed variation of each rotor. In standard commercial vehicles, the structure (called frame) is commonly composed of a center where the electronic components and power source are mounted, and several arms that extend radially from that center, at which end the rotor is placed. These arms have all the same length and are uniformly distributed in a circle. Generally, a four-rotor aircraft (quadrotor) is enough for most basic personal and commercial applications, including good quality aerial photography and filmmaking with small cameras, but when heavier payloads are required or more stability is needed, vehicles with six and eight rotors are a preferred choice. Other common configurations include those where each arm of the frame has two coaxial rotors, both generating thrust in the same direction but with propellers rotating in opposite directions. In this way, the weight of the arms is reduced, but the combined thrust is not twice that of one single motor due to aerodynamic effects. These alternative multirotor configurations are shown in Figure 1.

Multirotors do not rely on complex mechanics to maneuver, but instead are based on speed (and thus force) variation of the set of motors. Suppose a standard hexarotor, with six arms of the same length, uniformly distributed in a circle. Then, the individual actions of the rotors to perform basic maneuvers are described in Figure 2. Consider a reference frame fixed to the vehicle (body frame), where the origin is at the center of the vehicle coincident with the center of mass, the x axis points to the front of the vehicle, the y axis to the right, and the z axis downwards. If the rotors are generating all the same thrust, and the total force is equal to the weight of the vehicle, then no torque is exerted in any of the three axes, and the vehicle remains still in the air. If all the rotors' forces are increased or decreased equally as in Figure 2.a, then the vehicle will ascend

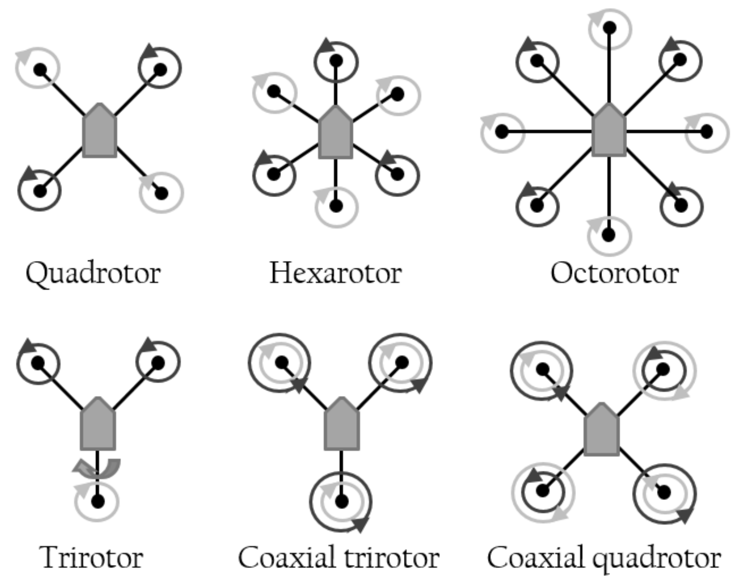

Figure 1. Common types of multirotors.

or descend. To tilt the nose of vehicle downwards (over the y axis, defined as the *pitch* angle), the rear rotors should increase their force while the front ones should decrease them, as shown in Figure 2.b. Analogously, to tilt the vehicle sideways (over the x axis, defined as the *roll* angle), the right (or left) rotors should increase their force while the left (or right) ones should decrease them, as shown in Figure 2.c. A rotation over the z axis, called a maneuver in yaw, is also exerted by rotor speed variation. As there is a rotor with a propeller rotating at the end of each arm, by conservation of angular momentum there appears an opposite torque in the frame over each rotor's axis, coincident in this case with the z axis of the vehicle. If all propellers rotated in the same direction, that would result in an uncompensated torque in the z axis that could not be controlled. The solution is to use two types of propellers, called CW and CCW (clockwise and counter-clockwise rotating propellers), which are identical in their construction, but generate thrust in the same direction while rotating in opposite directions. Then, CW and CCW propellers are used (generally) alternately in the vehicle's rotors, and a yaw maneuver can be performed by increasing the speed of the CW (or CCW) rotors, while decreasing the speed of the CCW (or CW) ones.

The yaw axis in standard multirotors is the one in which it is, in some sense,

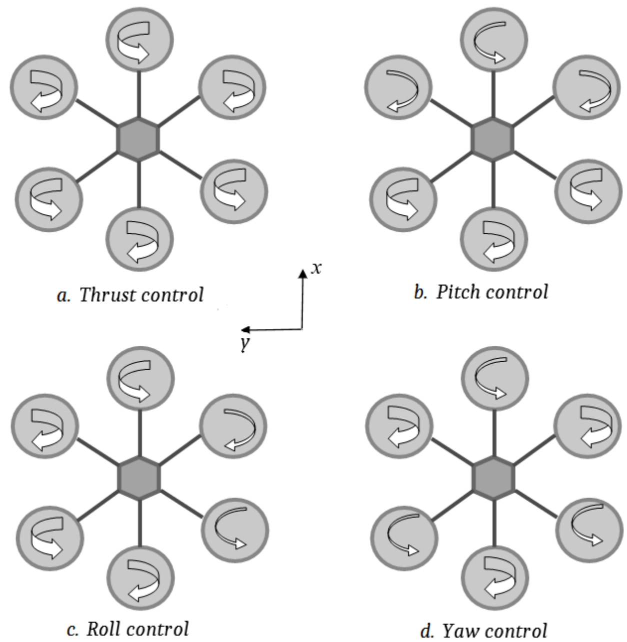

Figure 2. Multirotor control mechanics.

most difficult to exert torque. This is because the torque produced in the z axis due to the lightweight spinning propeller is significantly lower than the torque in the other two axes, between 10 and 50 times, according to experimental tests [4, 5]. This will present an important limitation when dealing with failures in rotors, as, usually, it is the yaw axis the one that restricts the most the maneuverability of the system.

To provide a better understanding of how the maneuvers described in Figure 2 are performed, a brief description of multirotors mechanics and dynamics is presented next. Modern multirotors consist of three important groups of components: the frame, the flight computer, and the power system. In general, the frame is the mechanical structure over which all the rest of the components are mounted, and it is generally fixed, but may also have mobile parts (e.g., in trirotors, servos to move the rotors are needed to control 4 DOF [6, 7]). The flight computer comprises all the electronics (except for the actuators) that are mounted on the vehicle: the necessary sensors (Inertial Measurement Unit (IMU), compass, barometer, GPS, among others), a microcontroller that runs

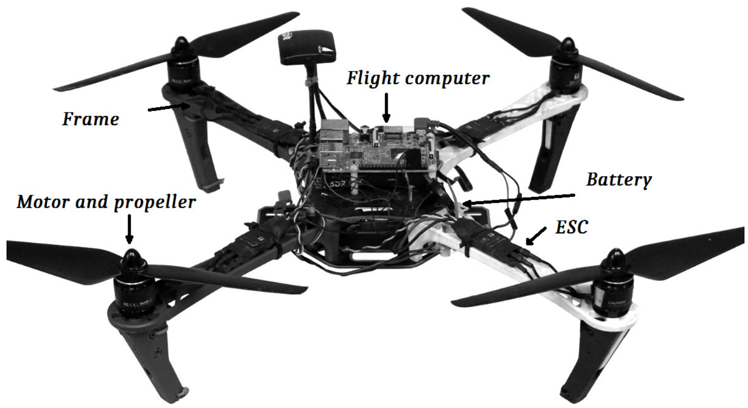

Figure 3. Main parts of a multirotor aircraft.

the algorithms for position and attitude estimation, the control algorithm, and additional tasks, and other components such as voltage regulators, visual and sound indicators, and radio-control receivers. Finally, the power system is composed of a main power source (LiPo and LiIon batteries most frequently used), and the actuators. Each actuator set includes a motor, usually of the BLDC type (brushless, DC powered, synchronous), a propeller, commonly of the fixed or foldable type (in rare cases with variable pitch), and an electronic speed controller (ESC) which allows to convert DC voltage to an AC signal, in order to control the speed of the motor. An example of the components for a standard quadrotor is shown in Figure 3.

In Figure 4, a simplified diagram of the data acquisition and control processes in a typical multirotor system is shown. The flight computer uses the information acquired by the different sensors into a fusion algorithm that outputs the best estimation of the orientation and position of the vehicle, in order to be used by the control system. Generally, two different control modes are used to operate a vehicle: commanding directly its orientation and vertical thrust, or generating a position reference that the vehicle has to follow.

In the first case, the orientation reference is compared with the estimation, and, if a difference exists, the orientation controller (commonly a PID) outputs a torque $q \in \mathbb{R}^3$ in order to correct the deviation. This torque is converted into a set of forces to be commanded to each of the rotors of the aircraft by the allocation algorithm.

For the position control case, an outer control loop is added, where the posi-

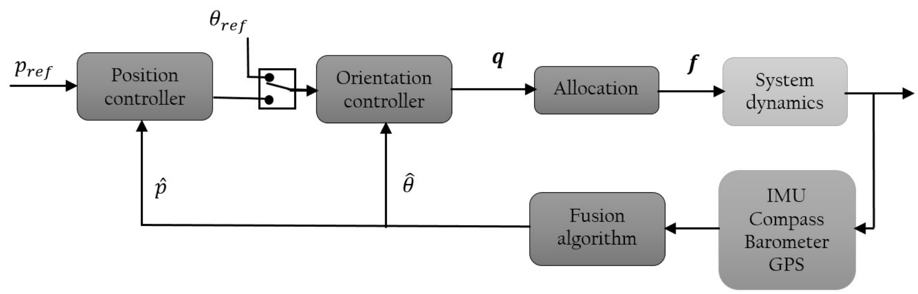

Figure 4. Simplified typical data acquisition and control diagram of a multirotor vehicle.

tion reference is compared with the estimation, and another controller computes the necessary maneuver to correct deviations, which is then commanded to the inner loop controller.

1.2. Notions of Fault Tolerance and Safety Actions

Fault tolerance is the ability of a system to continue operating properly in the event of one or more failures in some of its components. The key to define fault tolerance in multirotor aerial systems is how *proper operation* is defined for a given situation, and how is the system's degradation measured in the event of a failure. Based on this, different strategies can be adopted, being the following some of the most common.

In case of a failure, the vehicle is able to:

a) Fall to the ground in a safe manner to prevent harm to third parties (e.g. ultralight vehicle, or by means of a sound alarm that alerts of the falling vehicle).

b) Fall to the ground in a safe way to prevent harm to others and to itself (e.g. deploying a parachute or similar solution).

c) Descend and land in a controlled way when losing part of the sensor information, or of the control of the vehicle (e.g. failures of non-critical sensors, rotor failures in over-actuated systems).

d) Continue flying, and modify the flight plan according to the degraded capabilities of the system (e.g. shorten the flight plan, avoid certain objectives).

e) Continue flying normally (only for cases when the failure does not affect the performance of the vehicle).

Item **(a)** is the minimum allowed degree of safety for responsible users to operate this kind of aircraft, in order to minimize possible damage. Generally, these aircraft are used either away from populated areas where a falling vehicle would not cause damage to other people (hobbyist flights, aerial photography of landscapes, etc.), or in zones where the attendees are aware of the risks involved and adequate security measures are taken (drone racing, professional aerial filming, etc.). Still, collateral damages to wildlife or wildfire ignitions are within the possibilities. In some cases, this is the only possible solution to deal with some kind of failures. For example, a total failure in the power system (i.e. the battery, due to a burnt wire, short circuit, or other) would leave the vehicle completely unresponsive, falling immediately to the ground. Then, the only option to minimize damages is if the vehicle is light enough not to cause damage on impact, and/or try to warn bystanders using a self-powered sound alarm.

Similar cases to **(a)** are considered in the safety action taken in **(b)**. That is, the vehicle is still going to fall to the ground due to a failure, but further measures can be taken in order to guarantee the integrity of the vehicle and its payload, while also decreasing the risks of collateral damage. Parachute systems are a reasonable way to provide additional safety, at the cost of an increased weight, proportional to that of the vehicle [8]. This may, in turn, limit the allowed weight of the payload, or decrease the overall flight time. On the other hand, this provides a fast-deploying safety measure that can handle any type of failure of the vehicle, as long as the parachute activation system is independent from the main battery, to cover a possible failure in it. The vehicle should still fall to the ground without any direction control, but at a greatly reduced speed, giving more time for bystanders to move away, and decreasing the destruction on impact.

Item **(c)** considers some cases where, while the failure may be severe, there is a safe way to land the vehicle without major difficulties. For example, a vehicle that loses all position information (frequently GPS systems) or heading information (compass) while following a preset path, cannot continue the mission, and manual control has to be taken by the pilot to return the vehicle and land it safely. Another case, frequent in over-actuated systems, is a failure of a rotor that causes to lose control over some, but not all, degrees of freedom (DOF) of the vehicle. This is a case similar to a loss of the tail rotor of an helicopter, but, in the case of multirotors, while the vehicle starts to spin over the vertical axis uncontrollably, still allows position control while remaining stabilized, letting a pilot land it safely.

In **(d)**, it is considered that the vehicle suffers a failure in a component, and somehow its capabilities are degraded, but is still able to fly. For example, a vehicle that uses as a power source two batteries in parallel and suffers a failure in one of them, may still continue with its preset path or mission, but sees its flight time severely reduced, which may prevent it to complete the mission. This is also the case for vehicles for which rotor disposition allows the loss of a rotor while not losing degrees of freedom, but at a cost of a reduced maneuverability. Ideally, the vehicle should, as soon as a failure occurs, return to a safe point and land, but there are nowadays applications where the completion of the mission may be critical, as happens in medical and urgent aid related missions.

Finally, fault tolerance as defined in **(e)** considers the cases where a failure in a component is transparent from the point of view of the mission. For example, as IMU, digital compass, and other relevant sensors have a negligible weight in the vehicle, redundant sensors may be mounted within the flight computer, using one or another depending on the health of each [9, 10]. Thus, in case of a failure in one of them, the system could switch to another sensor, and the failure would pass as unnoticed from the mission point of view.

2. Advances in Multirotor Design and Fault Tolerance

Through the years, there have been many contributions that dealt with varied multirotor designs to approach several issues regarding both nominal operation and fault situations. As mentioned before, in a nominal condition, it is desired to control the aircraft in 4DOF, to be able to exert torques around the vehicle's three axes, and force in the vertical axis; then, the movement in the horizontal plane is accomplished by pitch and roll maneuvers, which is the working principle of standard multirotors with four or more rotors. Moreover, when six or more rotors are used, it is possible to control independently position and attitude (i.e. 6DOF).

In [11], the authors present a modification to the standard hexarotor design, by tilting the rotors' axis inwards (towards the center of the vehicle) to achieve a fully actuated vehicle, while in [12] it is also considered tilting the rotors sideways (along the arms' axis), where an optimal disposition of the rotors is obtained for a case where a desired trajectory is to be followed with the minimum control effort. The work done in [13] proves that almost any non-planar hexarotor structures (those where the rotors are not distributed in a co-planar way) can be approximated by a planar arrangement, by an adequate orientation of each rotor, thus reducing the problem of hexarotor design to the orientation of each rotor-propeller set. Also, it is shown that the inwards and sideways tilting angles may be optimized to obtain the best behaviour in translational and/or rotational dynamics.

Different overactuated hexarotor designs were proposed to achieve 6DOF control, such as those presented in [14, 15], where a servomotor is added to a hexarotor to tilt all the rotors sideways simultaneously, and the tilting angle can be optimized from the point of view of energy efficiency to follow a given trajectory.

In the works mentioned above, all the rotor-propeller sets are considered to be unidirectional, that is, the actuators can only exert force in one direction. Then, stating that such vehicles are capable of 6DOF control is not accurate, as the vehicles cannot exert force downwards in the vertical axis, but rather rely on the existence of the gravity force. This means that a vehicle is not controllable if it is turned upside down. To achieve true 6DOF control, it is proved in [16] that seven is the minimum amount of unidirectional rotor-propeller sets needed, and an example configuration of the rotors to achieve it is given. Other approaches to this issue are, for example, an octorotor with bidirectional actuators in a cube-like disposition [17] and in a rod-like disposition [18], and a twelve-rotor, six-arm vehicle with coaxial unidirectional actuators, that can be dynamically rotated over the arms' axes using servomotors [19].

Another advantage of over-actuated vehicles is that they could be able to compensate for the failure of one of the rotors with the action of the remaining ones, making possible to fly with a vehicle that does not lose maneuverability, in particular maintains its 4DOF (attitude and altitude), even when one of its rotors fails. Standard multirotors, even some over-actuated aircraft, are not always capable to maintain control in 4DOF with a rotor failing, leading to different approaches to design vehicles that consider this issue.

2.1. Rotor Fault Tolerance Design

Back to the topic of fault tolerant control in case of rotor failures, one may ask if any of the structures presented above represent a more convenient choice when dealing with rotor failures, whichever the nature of the failure may be.

For convenience in notation, when referring to the spinning direction of a rotor, a CW rotating rotor will be defined as a **P** rotor and a CCW one as an **N** rotor. Then, a PNPNPN hexarotor will represent a vehicle in which consecutive rotors have alternate spinning directions.

In [20], a thorough analysis is made over multirotor vehicles with six or eight unidirectional actuators, distributed in a co-planar way over a regular polygon, with all their axes perpendicular to this plane, considering the attainable torque set for a hovering state. It is shown that PNPNPN hexarotors cannot maintain 4DOF control in case of a total failure (impossibility to exert thrust) in any of its rotors, but a PPNNPN design allows for 4DOF control only in the cases of a fault of this kind in the first four rotors, and even in a particular case of two rotors in failure. Also, it is proved that an octorotor vehicle, both for a PNPNPNPN and a PPNNPPNN configuration, does not lose control over any DOF in case of a single failure in a rotor, and even most cases of two failures. A more thorough analysis of the PPNNPPNN octorotor planar vehicle can be found in [21], where the control allocation problem is analyzed for particular cases of simultaneous failures in 1, 2, 3 and 4 rotors.

Other complex mechanical solutions are proposed in the literature to address total rotor failures, as the one proposed in [22], where a fault tolerant quadrotor structure is proposed by adding servomotors to reconfigure the position and orientations of the rotors in-flight, which rendered it capable of rejecting perturbations even in the event of a failure in two of its rotors.

Another kind of more relaxed solutions for cases of rotor failures in multirotors are those in which the vehicle relinquishes control over one DOF, but still being able to fly in a predictable way. One example of this solution in the one presented in [23], where a standard PNPNPN hexarotor gives up control of the yaw angle in order to maintain control over pitch, roll, and vertical force, allowing it to hover in a static position while spinning uncontrollably in the vertical axis. As mentioned before, the torque in the z axis is more limited with respect to the other axes due to the nature of the maneuver (also, the moment of inertia in this axis tends to be higher). Moreover, maintaining pitch, roll and vertical force control allows to control the position of the vehicle without restrictions, thus permitting to land it safely. Another example is shown in [24], where a quadrotor is considered tolerant to a failure in 1, 2 or 3 rotors, if it is able to fly in the vicinity of a position reference point. This same principle is applied to the design of a single-rotor vehicle in [25]. However, as typical commercial systems are designed such that the weight of the vehicle is half of the maximum thrust provided by the set of actuators (for maximum maneuverability), this solution is not always feasible when there exist failures in at least half of the rotors.

There are different works that deal with rotor failures different to that of an incapability to exert thrust. In [26], analysis of a blockage in a rotor (locked at a given speed) yields that a quadrotor is not able to control 4DOF, but both a PNPNPN and a PPNNPN planar hexarotor are. In [27], an incremental backstepping controller is implemented to deal with modelling errors of the vehicle, as well as degradation in the efficiency of the rotors.

In [28] a hexarotor structure was proposed, consisting of a standard PNPNPN planar distribution where the rotors are tilted all at the same (fixed) angle towards or away from the center of the vehicle. Experimental validations of this result can be found in [29], where an inwards-tilted hexarotor is capable of hovering with limited maneuvering, and in [30], where instead of a zero pitch and roll hovering state, an optimal orientation of the vehicle is found in order to achieve a static flight position. Results presented in [28], were extended in [31] to obtain a vehicle independently controllable in 6DOF, even in case of any rotor failure.

The remainder of the chapter deals exclusively with rotor failures, particularly when a rotor loses all capability of exerting thrust and torque. Moreover, a multirotor aerial system will be considered fault tolerant only if, in case of a failure of a rotor, is still able to control independently attitude and altitude (4DOF).

In the following section we present a simple model for the unmanned multirotor system, which is useful to illustrate how it is possible to study the fault-tolerant control problem as a simple algebraic problem.

2.2. Vehicle Model

In a normal state of operation, each unidirectional rotor-propeller set produces a force $f_i \in [0, F_M]$, being F_M the maximum force at top speed. In practice, each motor's speed is commanded through a Pulse Width Modulated (PWM) signal u_i, which takes a value between 0 and 100%. Near the nominal operating point, a linear relation between the PWM signal and the exerted force is assumed, with $f_i = k_f u_i$. It is also considered that each motor exerts a torque on its spinning axis, $m_i = \pm k_t u_i$, where the sign depends on the spinning direction (CW or CCW), which may also be expressed as $m_i = \pm(k_t/k_f)f_i$. The k_f and k_t constants are usually established experimentally.

The total vehicle force $f \in \mathbb{R}^3$ and torque $q \in \mathbb{R}^3$ in the body frame coordinates (see Figure 5) satisfies the following equations:

$$f = k_f E u, \qquad q = (k_t E J + k_f H)\, u \tag{1}$$

$$E = [e_i^c]_{i=1,n}, \qquad H = [d_i^c \times e_i^c]_{i=1,n} = [h_i^c]_{i=1,n}, \tag{2}$$

Here, the location of the center of mass of the i–th motor is given by the column vector $d_i^c \in \mathbb{R}^3$, and the direction of the corresponding force is given by the column vector $e_i^c \in \mathbb{R}^3$. Both vectors are represented in body frame coordinates. J, is a diagonal matrix with diagonal entries $j_{ii} = (-1)^{i+1}$, for $i = 1, ..., n$, for a PNPNPN configuration.

Let $A = (k_t E J + k_f H)$, given a desired torque q, the control allocation algorithm finds (if there exists) $u \geq 0$ (with non-negative components) such that $q = Au$. The following theorem gives necessary and sufficient conditions for the existence of solutions of this problem [32].

Theorem 1. *Let $A \in \mathbb{R}^{3\times n}$. The following conditions are equivalent:*

1. *For each $q \in \mathbb{R}^3$ there exists $u \geq 0$ such that $q = Au$.*
2. *Matrix A has full rank and there exists $w \in \mathbb{R}^n$ with strictly positive components, i.e., $w > 0$ such that $Aw = 0$ i.e., w belongs to the kernel of A (denoted $ker(A)$).*

The proof this theorem gives an idea of how to find the solutions. First, suppose that Item 1. holds, and let $v > 0$ be an arbitrary vector with strictly positive components. Since for every $q \in \mathbb{R}^3$, there exists a vector $u \geq 0$ such that $Au = q$, in particular it is possible to find $u \geq 0$ such that $-Av = Au$, then $w = v + u \in \mathbb{R}^n$ is a vector with strictly positive components such that $Aw = 0$. On the other hand, if A has full rank, then for every $q \in \mathbb{R}^3$, there exists $v \in \mathbb{R}^n$ such that $Av = q$. If $w > 0$ satisfies $Aw = 0$, it is possible to find a number $0 < \alpha \in \mathbb{R}$ such that $\alpha w + v$ is a strictly positive vector, and $A(v + \alpha w) = q$.

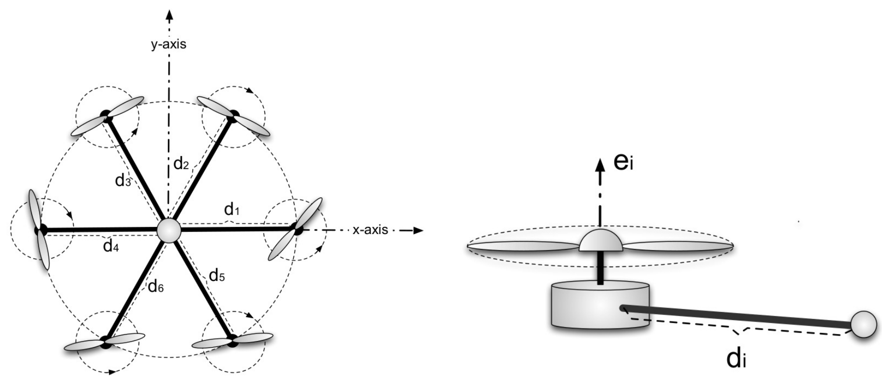

Figure 5. Hexacopter axes in standard configuration.

The positive vectors $w > 0$ in the kernel of A, allow to construct the PWM signals $u \geq 0$ such that achieve the desired torque. Analyzing the rank and kernel of A, it is possible to see if with the given actuators' disposition, it is possible to achieve any desired torque.

2.3. Standard Hexacopter Configuration

Consider the vehicle in Figure 5, where all rotors are identical and their thrust and torque are exerted in the direction of the vehicle's z axis. In this case we have:

$$d_i^c = \ell \begin{bmatrix} c\alpha_i \\ s\alpha_i \\ 0 \end{bmatrix}, \qquad e_i^c = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad h_i^c = \begin{bmatrix} \pm\ell s\alpha_i \\ \mp\ell c\alpha_i \\ 0 \end{bmatrix} \tag{3}$$

where $\alpha_i = (i-1)\frac{\pi}{3}$ rad, $i = 1, ..., 6$ and $\ell > 0$ is the distance to each vertex of the hexagon. As a consequence,

$$A = \begin{bmatrix} k_f\ell & 0 & 0 \\ 0 & k_f\ell & 0 \\ 0 & 0 & k_t \end{bmatrix} \begin{bmatrix} 0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & 0 & -\frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ -1 & -0.5 & 0.5 & 1 & 0.5 & -0.5 \\ 1 & -1 & 1 & -1 & 1 & -1 \end{bmatrix}. \tag{4}$$

Notice that, $A \in \mathbb{R}^{3\times 6}$ has full rank and the vectors w in kernel of A can be written as $w = \begin{bmatrix} \alpha & \beta & \gamma & \alpha & \beta & \gamma \end{bmatrix}^T$, with $\alpha, \beta, \gamma \in \mathbb{R}$. Taking $\alpha, \beta, \gamma >$

0, from Theorem 1 it follows that a standard hexarotor aircraft can reach any desired torque.

If we assume that one of the rotor fails, for instance rotor 2, the force excerted by this rotor is zero. In this condition, if we want to study if the aircraft can reach any torque, we need to study if there is a vector in $ker(A)$ of the form $w = \begin{bmatrix} w_1 & 0 & w_3 & w_4 & w_5 & w_6 \end{bmatrix}^T$ with components $w_j > 0$, for $j = 1, 3, 4, 5, 6$.

With this idea, it is easy to prove that we need at least six rotors to have a fault-tolerant vehicle, because if $x, y, z \in \mathbb{R}^n$ are vectors such that $x_i = 0$ and $x_j > 0$ (for every $j \neq i$), $y_k = 0$ (for some $k \neq i$ and $y_j > 0$, for every $j \neq k$) and $z_l = 0$ (for some $l \neq i, l \neq k$ and $z_j > 0$, for every $j \neq l$), then x, y, z are linearly independent vectors. In fact, let $N = \begin{bmatrix} x & y & z \end{bmatrix}$ be the matrix containing vectors $x, y, z \in \mathbb{R}^n$ as columns. Observe that, there exists a row-permutation matrix $U \in \mathbb{R}^{n \times n}$ such that

$$M = UN = \begin{bmatrix} 0 & m_{12} & m_{13} \\ m_{21} & 0 & m_{23} \\ m_{31} & m_{32} & 0 \\ m_{41} & m_{42} & m_{43} \\ \vdots & \vdots & \vdots \\ m_{n1} & m_{n2} & m_{n3} \end{bmatrix},$$

with $m_{ij} > 0$. Since

$$det \begin{bmatrix} 0 & m_{12} & m_{13} \\ m_{21} & 0 & m_{23} \\ m_{31} & m_{32} & 0 \end{bmatrix} > 0,$$

it follows that the columns of M are linearly independent vectors and then also are columns of N because U is invertible. So, if we want a matrix $A^{3 \times n}$ such that A is full rank and vectors $x, y, z \in ker(A)$, then $n \geq 6$. The question is if with six rotors is enough.

2.4. Fault-Tolerant Hexacopter Design

Observe that, if one motor fails, matrix A loses one of its columns, the one corresponding to the failed rotor. Suppose that $A_j \in \mathbb{R}^{n \times 5}$ is matrix A after removing column j. For instance,

$$A_2 = \begin{bmatrix} k_f\ell & 0 & 0 \\ 0 & k_f\ell & 0 \\ 0 & 0 & k_t \end{bmatrix} \begin{bmatrix} 0 & \frac{\sqrt{3}}{2} & 0 & -\frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ -1 & 0.5 & 1 & 0.5 & -0.5 \\ 1 & 1 & -1 & 1 & -1 \end{bmatrix}. \qquad (5)$$

So, to verify if the aircraft can be fully controllable in attitude if motor 2 fails, it is necessary to study if there exists a strictly positive vector in $ker(A_2)$, or analogously, if there is a vector in $ker(A)$ of the form $w = \begin{bmatrix} w_1 & 0 & w_3 & w_4 & w_5 & w_6 \end{bmatrix}^T$ with components $w_j > 0$, for $j = 1, 3, 4, 5, 6$.

Observe that matrix A_2 is a full rank matrix, but vectors $s \in ker(A_2)$ are of the form $s = \begin{bmatrix} \alpha & \beta & \alpha & 0 & \beta \end{bmatrix}^T$, where $\alpha, \beta \in \mathbb{R}$; since there is no vector in $ker(A_2)$, with strictly positive components, then the standard hexacopter is not fault tolerant if motor 2 fails. In fact, it is easy to see that it is not fully controllable if any of its motors fails. The vehicle can not reach torques in a particular direction. To overcome this limitation, a tilted-rotor hexacopter design proved to be useful.

Tilt-rotor aircraft have been widely used for different reasons, for improving the maneuverability [12, 14], power efficiency [33] or fault tolerance [28, 30, 31, 34]. In the next section, based on Theorem 1, it is shown that by tilting the rotors (or arms) a fixed-angle, it is possible to design a hexarotor vehicle capable of preserving 4DOF if any of its motors fails. We start with the simplest design and then we summarize the most relevant generalizations of this design. We also present some limitations and appropriate solutions to overcome these limitations.

2.4.1. Tilted Configuration

Since the probability of failure is the same for each motor, it is reasonable to propose a symmetric configuration as shown in Figure 5. Consider a design where the rotors are tilted a fixed angle θ as shown in Figure 6, with the tilt angle being the same for all rotors ($i = 1, ..., 6$). The standard configuration is a particular case, where $\theta = \pi/2$. As a consequence, the matrix that relates the PWM signals u with the torque q is given by $A = A(\theta)$,

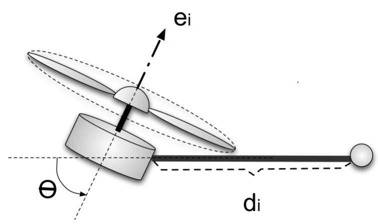

Figure 6. Tilted motor configuration.

$$A = k_t \begin{bmatrix} -c\theta & \frac{1}{2}\left(c\theta + \frac{\sqrt{3}\ell k_f s\theta}{k_t}\right) & \frac{1}{2}\left(c\theta + \frac{\sqrt{3}\ell k_f s\theta}{k_t}\right) & -c\theta & \frac{1}{2}\left(c\theta - \frac{\sqrt{3}\ell k_f s\theta}{k_t}\right) & \frac{1}{2}\left(c\theta - \frac{\sqrt{3}\ell k_f s\theta}{k_t}\right) \\ -\ell\frac{k_f}{k_t}s\theta & \frac{1}{2}\left(\sqrt{3}c\theta - \frac{\ell k_f s\theta}{k_t}\right) & \frac{1}{2}\left(\frac{\ell k_f s\theta}{k_t} - \sqrt{3}c\theta\right) & \ell\frac{k_f}{k_t}s\theta & \frac{1}{2}\left(\sqrt{3}c\theta + \frac{\ell k_f s\theta}{k_t}\right) & \frac{1}{2}\left(-\sqrt{3}c\theta - \frac{\ell k_f s\theta}{k_t}\right) \\ s\theta & -s\theta & s\theta & -s\theta & s\theta & -s\theta \end{bmatrix} \tag{6}$$

where $c\theta = cos(\theta)$ and $s\theta = sin(\theta)$. It is not hard to see that A has full rank and $w = \begin{bmatrix}1 & 1 & 1 & 1 & 1 & 1\end{bmatrix}^T \in ker\,(A)$, for any $0 \leq |\theta - \pi/2| < \pi/2$. So, according to Theorem 1, the tilted-rotor hexacopter is fully controllable if every rotor is working properly. But, in this case, it is also fully controllable after a failure in any of its rotors. Suppose that rotor 2 fails, it is easy to prove that A_2 has full rank. In fact,

$$\det\left(A_2 A_2^T\right) = \frac{27}{4} k_t^2 sin^2(\theta)\left[cos(2\theta)\left(k_t^2 - \ell^2 k_f^2\right) + \ell^2 k_f^2 + k_t^2\right]^2 \neq 0, \tag{7}$$

for every $0 \leq |\theta - \pi/2| < \pi/2$. Furthermore, let $w = \begin{bmatrix} w_1 & w_2 & w_3 & w_4 & w_5 \end{bmatrix}^T$, with:

$$w_1 = \frac{1-\alpha}{\alpha+1} - w_4\frac{1-\alpha}{2\alpha} \tag{8}$$
$$w_2 = 1 \tag{9}$$
$$w_3 = 1 - \frac{1-\alpha}{2\alpha}w_4 \tag{10}$$
$$w_5 = w_4 + \frac{1-\alpha}{\alpha+1}, \tag{11}$$

where $\alpha = \frac{k_t}{\sqrt{3}\ell k_f \tan\theta}$ and $|\alpha| < 1$. This vector $w \in ker(A_2)$ and, if $0 < w_4 < |2\alpha/(1+\alpha)|$ it follows that $w > 0$. Then, for a symmetric and tilted configuration of rotors with angle $\theta \neq \pi/2$ and $|\tan\theta| \neq \frac{k_t}{\sqrt{3}\ell k_f}$, the hexacopter can reject perturbation torques in any direction in $\mathbb{R}^3$ in order to maintain its attitude, even with the failure of one of its rotors. Also, observe that $|\alpha| \to 0$, *i.e.*, $\theta \to \pi/2$, is a desired condition since it maximizes thrust. In order to compute a practical value for the tilt angle θ, also the vehicle's thrust should be considered.

2.5. Thrust Equations

Let $v > 0$ be the thrust of the hexacopter, it can be computed as the sum of the forces in the z axis in body frame. Then, thrust v depends on the PWM signals $u \geq 0$, in the following way:

$$v = k_f \sin(\theta)\mathbf{1}^T u \qquad \text{with} \qquad \mathbf{1} = \begin{bmatrix}1 & 1 & 1 & 1 & 1 & 1\end{bmatrix}^T. \tag{12}$$

The mapping $u \to (q, v)$ is given by

$$\begin{bmatrix} q \\ v \end{bmatrix} = B(\theta)u = \begin{bmatrix} A(\theta) \\ k_f \sin(\theta)\mathbf{1}^T \end{bmatrix} u. \tag{13}$$

With the standard hexarotor configuration,i.e., with $\theta = \pi/2$, the vertical thrust is maximized. On one hand, as shown previously, fault tolerance cannot be achieved in this case. On the other hand, for every $0 < |\pi/2 - \theta| < \pi/2$ and $|\tan\theta| \neq \frac{k_t}{\sqrt{3}\ell k_f}$ it is possible to achieve any desired torque, with tolerance to one faulty rotor. It is expected to have a trade-off in the selection of θ, between the capability to reject torque disturbances and the ability to exert vertical thrust on the vehicle.

In order to address this issue suppose rotor number 2 is faulty, as before. The usual approach for the allocation of torque control commands q, if it exists, is to compute the actuator signal u of minimum norm. In case of failure in one rotor, there will be among all possible torque commands, a torque in a given direction that will more difficult to achieve. This particular *worst case* torque command (q_{wc}), whose direction induces a maximum over all minimum norm u actuator signals, will depend on the θ angle. In the case of $\theta = \pi/2$, as the torque gets closer to the worst case direction, the norm of vector u needed to allocate such a torque goes to infinity.

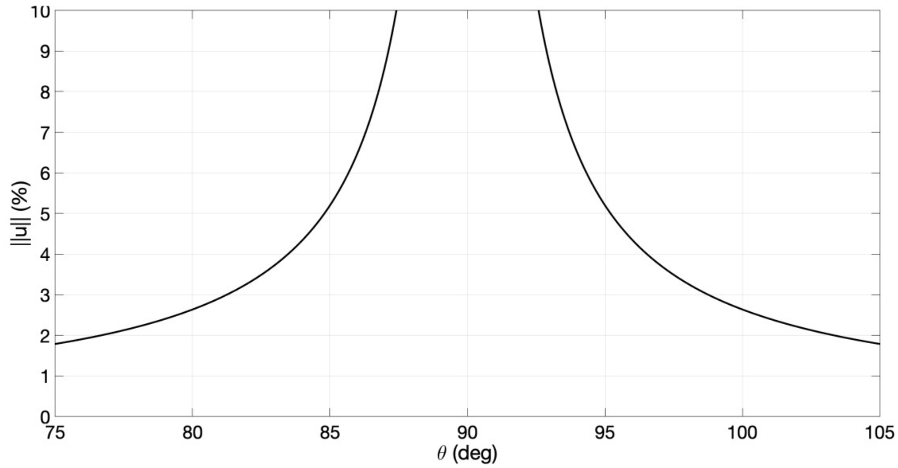

Figure 7. Minimum force for worst case torques.

Suppose that based upon practical considerations a given bound $q_{max} > 0$ is set on the torque commands whose allocation is sought. Within all torques $q \in \mathbb{R}^3$ with $\|q\| < q_{max}$, the following θ dependent function is proposed:

$$f(\theta) = \max_{\substack{q \in \mathbb{R}^3 \\ \|q\| < q_{max}}} \min_{\substack{B_j u = q \\ u \geq 0}} \|u\| \tag{14}$$

where $B_j = B_j(\theta)$ corresponds to matrix $B(\theta)$ with a failure in the j-th rotor, i.e., matrix $B(\theta)$ with column j removed.

For typical multirotors, the torques about the x and y directions are more important than those about the z direction. This is because angular accelerations about x and y change the vehicle's thrust direction, and therefore jeopardize position control [20, 23]. Thus, a weighted norm could be considered for $\|q\|$ in order to prioritize the x and y directions.

The objective is to compute the curve $f(\theta)$ as the one indicated in Figure 7 that plots the minimal motor forces $\|u\|$ needed to reject the worst case perturbation torques under motor failure.

The general idea is to determine a practical way to design the motor slant angle based on the worst case perturbation torque to be rejected and the minimum vertical thrust that maintains the hexacopter flying. From Figure 7 it can be observed that as θ approaches $\pi/2$ then the minimal force $u \geq 0$ needed to reach the worst case torque rapidly increases. On the other hand, as θ moves above or below $\pi/2$, the thrust is reduced according to $1/\sin(\theta)$. This establishes a compromise between the thrust reduction that can be afforded by tilting the rotors and the maximum perturbation torque that can be rejected after a failure of one rotor.

Although previous results provide a criteria to design the geometry of the vehicle analyzing the torque exerted, the fact that $u \geq 0$ does not consider the vehicle vertical thrust. In practice, this force $u \geq 0$ is chosen in such a way that it guarantees certain torque $q \in \mathbb{R}^3$ and vertical thrust $v > 0$.

2.5.1. Actuator Allocation

Assuming no rotor failures, in order to allocate a given pair torque/thrust $(q, v) \in \mathbb{R}^3 \times \mathbb{R}_+$, the actuators' signal $u \in \mathbb{R}^6$ is usually chosen as:

$$u_0 = B(\theta)^\dagger \begin{bmatrix} q \\ v \end{bmatrix} = \begin{bmatrix} A(\theta) \\ k_f s\theta \mathbf{1}^T \end{bmatrix}^\dagger \begin{bmatrix} q \\ v \end{bmatrix} \tag{15}$$

The reason to allocate u using the Moore-Penrose pseudoinverse $B^\dagger$, is that it renders the minimum norm solution. Other solutions based on generalized pseudo-inverses can improve the control allocation at the expense of a higher real time computational cost [35, 36].

Although it is possible to prove that for a given pair (q, v) there exists a positive solution $u \geq 0$ of Eq. (13) due to the existence of positive vectors in $ker\,(A(\theta))$, the positiveness of u_0 is not guaranteed. Let $C = A^\dagger$ and observe that, since $A = A(\theta)$ is full rank,

$$B(\theta)^\dagger = \begin{bmatrix} A \\ k_f s\theta \mathbf{1}^T \end{bmatrix}^T \left(\begin{bmatrix} A \\ k_f s\theta \mathbf{1}^T \end{bmatrix} \begin{bmatrix} A^T & k_f s\theta \mathbf{1} \end{bmatrix} \right)^{-1}. \tag{16}$$

Since $A\mathbf{1} = 0$ it follows that,

$$u_0 = \begin{bmatrix} A^T(AA^T)^{-1} & \frac{1}{6k_f s\theta}\mathbf{1} \end{bmatrix} \begin{bmatrix} q \\ v \end{bmatrix} = \begin{bmatrix} C & \frac{1}{6k_f s\theta}\mathbf{1} \end{bmatrix} \begin{bmatrix} q \\ v \end{bmatrix} \tag{17}$$

Then, a necessary and sufficient condition for the positiveness of u_0 is

$$\left| \min_{i=1,\ldots,6} (c_i^r)^T q \right| \leq \frac{v}{6k_f s\theta}, \tag{18}$$

where c_i^r is the $i-th$ row of matrix C. A consequence of Eq. (18), is that for each torque q, it gives a lower bound on the total thrust $v = v(\theta)$ in order to guarantee $u_0 \geq 0$. A more restrictive condition that simplifies the calculations would consist in a thrust that doesn't depend on q, *i.e.*, a thrust that assures $u_0 \geq 0$ in the worst case.

In [28] the following bound for the thrust is given. Let $0 < |\theta - \pi/2| < \pi/2$ and $q_{max} > 0$. Given a pair torque/thrust (q, v), if u_0 as given in Eq. (15), then $v > 0$ guarantees $u_0 \geq 0$ for every torque $\|q\| \leq q_{max}$ if and only if

$$v \geq k_f\, q_{max} \sqrt{\frac{(s\theta)^2(\ell^2 k_f^2 + 4k_t^2) + k_t^2(c\theta)^2}{\ell^2 k_f^2 k_t^2 (s\theta)^2 + k_t^4 (c\theta)^2}}. \tag{19}$$

This bound follows from a direct calculation of $\|c_i^r\|$. A similar bound can be obtained in the failure case.

When rotor j fails it is not as simple to find a condition as the one in Eq. (18) because $\mathbf{1} \notin ker\,(A_j(\theta))$. However, with additional calculations we can find similar conditions.

Suppose that $0 < |\theta - \pi/2| < \pi/2$, and define the following,

$$B_j(\theta) \;=\; \begin{bmatrix} A_j(\theta) \\ k_f sin(\theta)\mathbf{1}^T \end{bmatrix}, \qquad B_j^\dagger(\theta) = \begin{bmatrix} M & N \end{bmatrix} \tag{20}$$

for any $j = 1, ..., 6$, $M \in \mathbb{R}^{5\times 3}$, $N \in \mathbb{R}^5$ and $\mathbf{1} \in \mathbb{R}^5$. Given the pair torque/thrust (q, v), let

$$u_0 = B_j^\dagger(\theta) \begin{bmatrix} q \\ v \end{bmatrix}. \tag{21}$$

As a consequence, $u_0 \geq 0$ if and only if

$$M\,q + N\,v \geq 0. \tag{22}$$

As in the case without faulty rotors, a lower bound on $v > 0$ is sought, such that the existence of $u_0 \geq 0$ can be guaranteed for every $\|q\| < q_{max}$ with $q_{max} > 0$. The bound can be obtained, if the inequality

$$n_i\, v \geq q_{max}\, \|m_i^r\| \tag{23}$$

is satisfied for every $i = 1, ...5$, where m_i^r is the i-th row of M, and n_i the i-th element of vector N. Then, $v > 0$ guarantees $u_0 \geq 0$ for every torque $\|q\| \leq q_{max}$ if and only if

$$v \geq q_{max} \max_{i=1,\ldots,5} \frac{\|m_i^r\|}{n_i}. \tag{24}$$

Equation (24) provides a very practical design tool in order to determine the tilt angle θ based on minimum vertical thrust v and maximum perturbation torque q_{max}.

In [28] it was proved that $n_i > 0$ if and only if $0 < |\theta - \pi/2| < \pi/2$ and $|\tan\theta| \neq \frac{k_t}{\sqrt{3}\ell k_f}$. Furthermore, in [29] the numbers n_i and $\|m_i^r\|$ are computed as a function of θ. However, a numerical calculation of these values are enough in order to determine the optimal value of θ.

2.6. Experimental Validation

To provide experimental proof of the fault tolerant capabilities of an inward-tilted vehicle, a standard hexarotor model was used, weighing $3\,\mathrm{kg}$, with a rotor to rotor distance of $0.55\,\mathrm{m}$, and with actuator sets capable of exerting $1\,\mathrm{kg}$ of thrust, with a constant $k_t/k_f = 0.014$. The rotors were tilted inwards at an angle $\theta = 73°$. The experiment consisted in the vehicle taking off the ground, performing a number of maneuvers, and landing, with rotor 2 turned off during the entirety of the flight, which lasted around $100\,\mathrm{s}$.

The orientation of the vehicle during the flight is presented in Figure 8, where it is shown that the vehicle is able to follow the commanded references, which include impulses of high magnitude but short duration. However, it shows a mild oscillating behaviour, specially during aggressive maneuvers. In Figure 9, the PWM commands applied to the motors during the flight are shown, where it can be noted that rotor 5, the opposite to rotor 2, is working very close to its lower saturation limit. The PWM command for rotor 5 is saturated at 16%, as a lower value would turn it off (if the rotor is constantly turning on and off, it is working in a highly non-linear zone, which is desirable to avoid).

As the yaw angle torque is the most affected in case of a rotor failure, another experiment was carried out to analyze the performance in the z axis. In Figure 10, two opposite maneuvers in yaw are shown for the same vehicle, with rotor 2 turned off. The left one is performed by rotating the vehicle in the negative direction, maneuver which would require, in the nominal case, a reduction in the speed of rotor 2 (now turned off). In the failure case, it requires an increment in the speed of rotor 5, driving it away from saturation. The right plot of Figure 10 shows the opposite maneuver, rotating the vehicle in the positive direction, which would require an increase in the speed of rotor 2 in the nominal case, and in the failure case now pushes rotor 5 closer to saturation, presenting a worse behavior in the response, but still able to follow it.

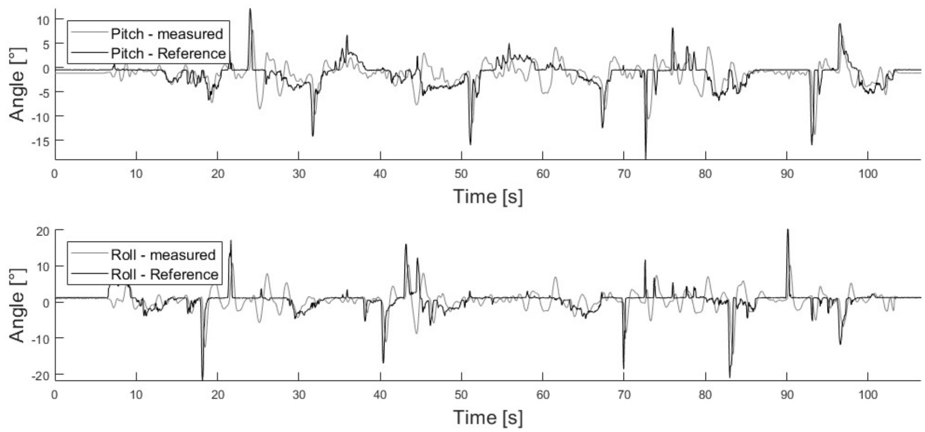

Figure 8. Orientation of the inwards-tilted hexarotor during a flight in which rotor 2 is turned off.

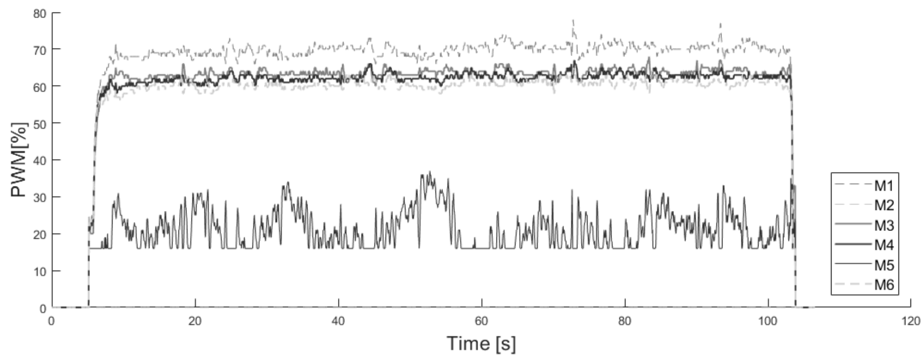

Figure 9. PWM commands of the inwards-tilted hexarotor during a flight in which rotor 2 is turned off.

2.7. Limitations of the Geometric Analysis

While the solution presented above is able to tell whether the vehicle is fault tolerant in case of a failure, it does not quantify how much its maneuverability is degraded.

Consider a standard hexarotor model, with the same physical characteristics as described above regarding size, weight and actuators, and suppose the motors are tilted inwards in order to have a fault tolerant vehicle. This time, the tilting

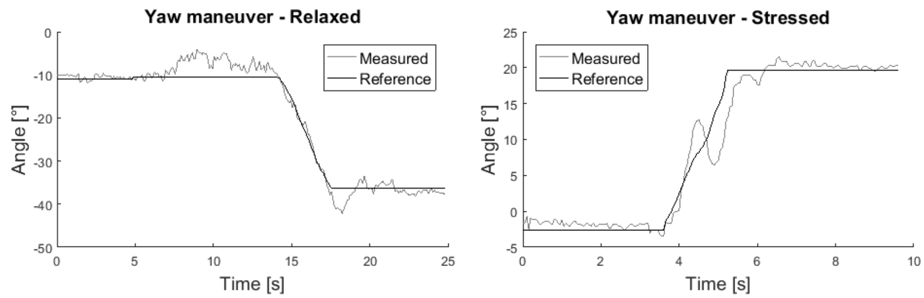

Figure 10. Yaw angle of the inwards-tilted hexarotor with rotor 2 turned off, during a yaw maneuver in the most relaxed direction (left) and in the most stressed direction (right).

angle is selected as $\theta = 65°$, allowing a 10% vertical thrust loss in the nominal case, a reasonable amount considering that the vehicle may carry an additional payload.

Considering that the vehicle is in hovering state (i.e. exerting a vertical force equal to its weight), Figure 11 shows the space of achievable torques both for the nominal and for a failure case in rotor 3. While the nominal vehicle shows a symmetric behaviour in its three axes (but with a ten times smaller torque in the z axis, as it is the axis in which it is most difficult to exert torque in), the vehicle with a failure shows a severe degradation in most directions, those in which the failing rotor has an appreciable contribution. As for the magnitude, the maximum torque that can be exerted in any direction, designated q_{max}, is $33.4 \times 10^{-3}\,\mathrm{kg\,m}$ for the nominal case, and $21 \times 10^{-6}\,\mathrm{kg\,m}$ for the failure case, three orders of magnitude lower. For the other cases of failure, the shape of the resulting achievable torque space is similar, but oriented in a different direction according to the rotor in failure.

The magnitude of q_{max} is given by a particular direction in the torque space, in which it is most difficult to exert torque in, and, both for the nominal and the failure case, is pointed almost parallel to the z axis. This suggests that q_{max} is highly influenced by the lower torque achievable in this axis, which is true for standard multirotor designs. A minimal value for this magnitude, in the considered vehicle, in order to achieve a reasonable flight performance under adverse weather conditions, is about $10 \times 10^{-3}\,\mathrm{kg\,m}$. Then, it is reasonable to assume that, in case of a failure, it will only be able to present an acceptable flight performance in controlled environments, and is not suitable for standard

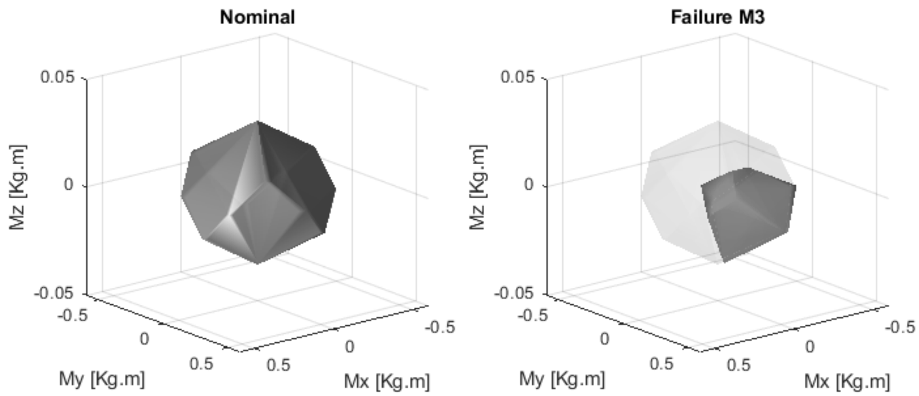

Figure 11. Achievable torque space for an inwards-tilted hexarotor in the nominal case (left), and for a case of a failure in rotor 3 (right).

outdoor flight missions.

3. IMPROVING THE MANEUVERABILITY IN CASE OF A FAILURE

At this point is notorious, for the proposed vehicles, the influence of the lower torque achievable in the z axis over the maneuverability of the vehicle when a failure occurs, and that the inwards-tilting solution, while theoretically valid, presents an unsuitable performance in real applications.

Regarding the experiments shown in Section 2.6, as rotor 2 (CW) is turned off, rotor number 5, which rotates in opposite direction (CCW), is forced to work near its lower saturation point. Increasing its speed would increment the torque it exerts in the z axis, but only two CW rotating rotors remain to compensate that increment. Then, if there existed a way in which the remaining CW rotors could increase the torque produced in the z axis, or in which all the CCW rotors decreased said torque, then it may be possible to drive the working point of rotor 2 away from its lower saturation.

Consider now that, besides the fixed inwards tilt angle, rotors may be tilted around the arms' axis at an angle δ, as shown in Figure 12. It will be considered that $\delta > 0$ corresponds to a tilt angle such that the torque exerted in the z axis of the vehicle is increased, with respect to $\delta = 0$. This also means that the sense

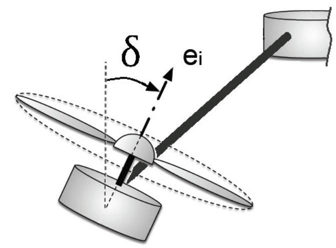

Figure 12. Side tilt angle δ. $\delta = 0$ is defined as the rotor pointing upwards, parallel to the vehicle's z axis, and $\delta > 0$ represents a tilt angle such that the torque the rotor exerts in the z axis is increased w.r.t. $\delta = 0$

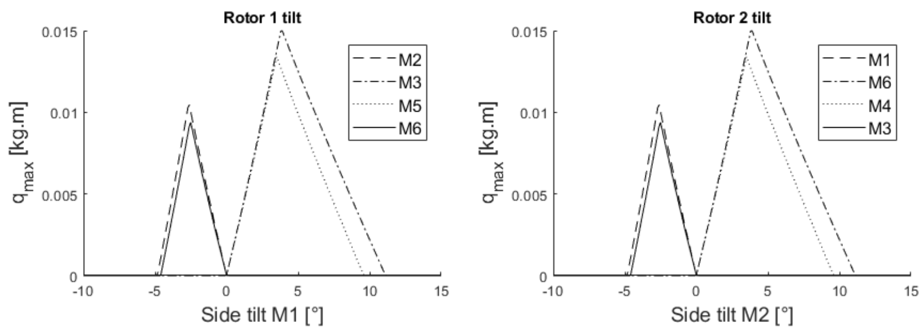

Figure 13. Maximum torque achievable in any direction in function of the tilt angle δ of rotor 1 (left) and rotor 2 (right), for each of the salvageable failures.

of rotation around the arm is different for CW and CCW rotors.

Going back to the proposed inwards-tilted hexarotor with $\theta = 65°$, suppose that one of the rotors is tilted at a fixed angle δ to analyze the behaviour of the vehicle in case of a failure. In Figure 13, it is shown how the q_{max} changes when tilting at a fixed angle only one of the rotors sideways, a CCW (rotor 1, left) or a CW (rotor 2, right), for each of the rotors in failure.

For rotor 1, a tilt angle $\delta_1 > 0$ improves q_{max} in cases of failure of rotors 3 and 5, which are all CCW rotating. In case one of the latter rotors fails, as rotor 1 can exert an increased torque in the z axis, it is able to compensate for the missing rotor by increasing slightly its speed. The opposite case occurs for a failure in rotors 2 and 6, which are CW rotating, as rotor 1 needs to lower the torque it exerts in the z axis, thus being better to tilt it at an angle $\delta_1 < 0$. As the vehicle is symmetric, a similar analysis is valid for a tilt in rotor 2, for a tilt angle $\delta_2 > 0$ for failures in rotors 4 or 6, and $\delta_2 < 0$ for failures in rotors 1 and 3. On the other hand, when a failure occurs in one of the rotors, the opposite tends to almost shut down, and is driven to a working point near the lower saturation limit. Hence, when rotor 4 fails, there is no noticeable improvement in q_{max} by tilting rotor 1, therefore it is not shown on the figure. An analogous situation occurs for a failure in rotor 5, as there is no improvement when tilting rotor 2.

The previous analysis shows that there does not exist an adequate fixed hexarotor structure to deal with all possible failures; but instead, if the system is able to actively change the sideways tilt of the rotors, it will be possible to improve its performance depending on which rotor fails. To accomplish this, at least two rotors should have tilting capabilities, and it should be implemented in a way such that one of them is CCW and the other CW, to achieve a better overall performance in case of failure. The tilting rotors cannot be placed opposite each other, as it is the case described above for a failure in rotor 4, unable to be compensated by tilting rotor 1. Therefore, in a PNPNPN hexarotor, the tilting rotors have to be placed in contiguous positions.

Consider that rotors 1 and 2 are selected to be actively tilted in-flight, then, for each of the possible failures, there is an optimal point to tilt either rotor 1 or 2 in order to obtain the highest q_{max}, being better to tilt rotor 1 for failures in rotors 3 and 5, and to tilt rotor 2 for failures in rotors 4 and 6 using $\delta > 0$. If rotor 1 or 2 fails, the only option is to tilt the remaining reconfigurable rotor at an angle $\delta < 0$.

Taking the best and worst cases presented above, in Figure 14, the achievable torque space is shown for the described hexarotor in case of a failure either in rotor 2 or 3, compensated by tilting rotor 1 adequately in order to obtain the best q_{max}. While the reconfigured fault tolerant solutions shows directions of preference when exerting torque, as the vehicle is still asymmetric, the volume of the achievable torque set is increased with respect to the non-reconfigurable case, as well as the magnitude of the torque achievable in all directions. The reconfigured system for a failure in rotor 3 shows a better performance than that for rotor 2, in accordance with Figure 13, obtaining a q_{max} of $14.7 \times 10^{-3}\,\mathrm{kg\,m}$ for the first case, and $9.3 \times 10^{-3}\,\mathrm{kg\,m}$ for the second. This represents a substantial increase in the maneuvering capabilities, rendering now the vehicle suitable for average outdoor flight conditions.

At this point, only the maneuverability of the vehicle in case of a failure has been analyzed, but in a real application, the system has to be able to detect a

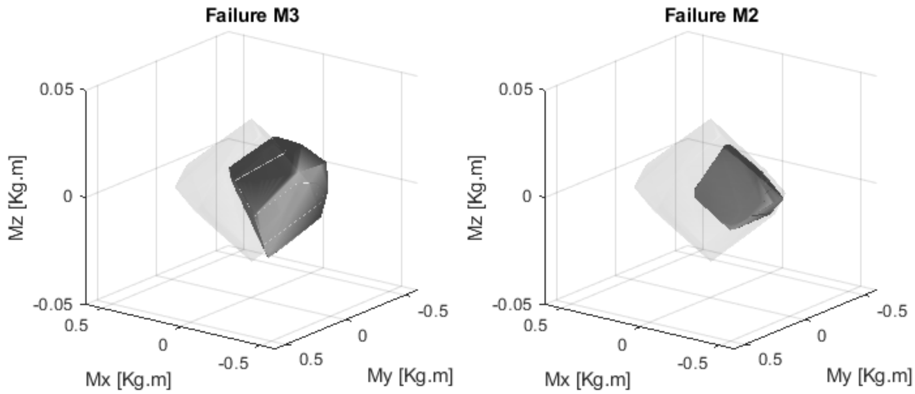

Figure 14. Achievable torque space for a reconfigurable hexarotor in case of a failure in rotor 3 (left) and for a case of a failure in rotor 2 (right). Rotor 1 is tilted at $3.8°$ and $-2.5°$ respectively, representing the best and worst cases for degraded maneuverability.

failure in a reasonable amount of time, and transition adequately between the nominal and the failure case. While the fault detection system is not analyzed here, previous results [37, 38] state that an adequate design for a similar system results in detection times under $400\,\mathrm{ms}$. To analyze the real performance of the reconfigurable hexarotor, an experiment was carried out for an inward-tilted vehicle, with a distance of $0.55\,\mathrm{m}$ between rotors, with a weight of $3\,\mathrm{kg}$, carrying actuator sets of $1\,\mathrm{kg}$ of maximum thrust force. A micro servomotor was placed in rotor 1 in order to tilt it sideways in-flight, as shown in Figure 15.

The experiment consisted in the vehicle taking off in nominal conditions, with all rotors working, going into a hovering state and, while the reference pitch, roll and yaw commands remained at zero, injecting a failure in rotor 3 (turning it off). After $400\,\mathrm{ms}$, rotor 1 is tilted to compensate the failure, the vehicle regains the hovering condition, and lands safely. Figure 16 shows the pitch, roll and yaw response of the vehicle for two identical experiments in an indoor environment, where the time axis is adjusted so that the failure is injected at t=$1\,\mathrm{s}$ and the system reconfigured at t=$1.4\,\mathrm{s}$. The vehicle is able to quickly recover, with both trials presenting an almost identical performance.

The PWM commands of the rotors are presented in Figure 17, for one of the trials, where before the failure all the rotors are operating in a similar working point, and after reconfiguration are still well away from both upper and lower

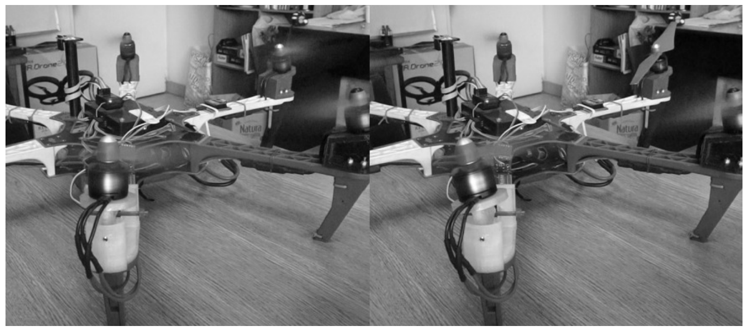

Figure 15. Inwards-tilted hexarotor with reconfigurable rotor 1.

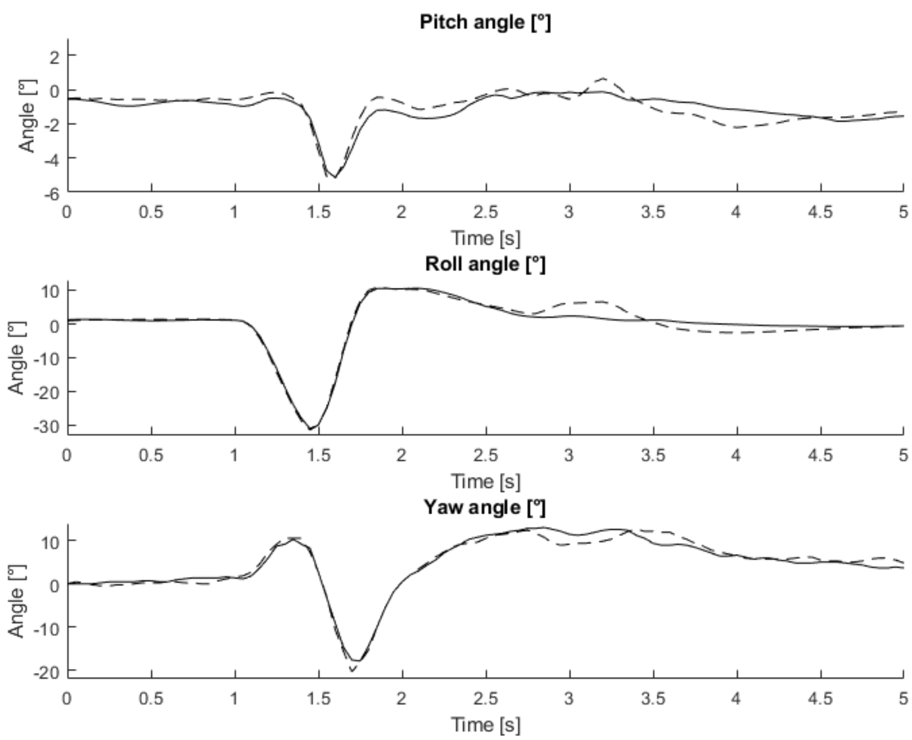

Figure 16. Angular response of an inwards-tilted hexarotor vehicle during two different experiments, where the fail is injected at t=1 s, and the vehicle is reconfigured after $400\,\mathrm{ms}$, maintaining all references at zero.

saturation limits, giving more room for speed variations to perform maneuvers.

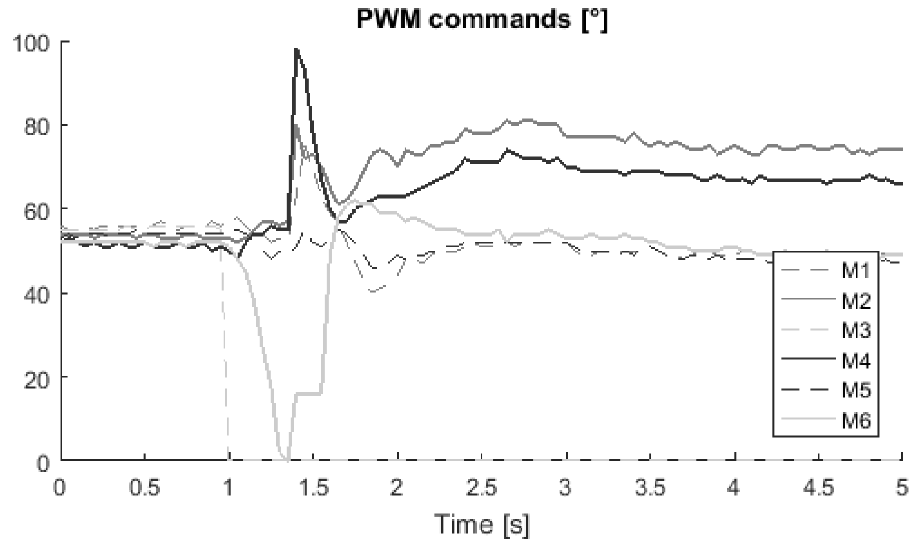

Figure 17. PWM command response of an inwards-tilted hexarotor vehicle corresponding to one of the reconfiguration delay experiments, where the fail is injected at t=1 s, and the vehicle is reconfigured after $400\,\mathrm{ms}$.

3.1. Rotor Fault Detection and Isolation

When dealing with failures in any kind of system, an important step prior to the adaptation of the system to the fault, is the detection of the existence of a failure, and the isolation of its cause. While direct methods can be applied, such as direct condition monitoring of the actuators through rotor power and/or speed sensing [39, 40, 41, 42], these are often not the preferred methods, as they require the use of specific rotors or speed controllers, or the addition of specific sensors for health monitoring.

A more common method to approach rotor fault detection is through a dynamical model of the vehicle, where it is represented by a body with a given mass and inertia tensor, affected by the forces and torques generated by the actuators. While this still requires an accurate modelling of the vehicle, as well as of the actuator set, it is more versatile and does not require additional electronics. This allows to predict accurately the vehicle's behaviour when a set of forces is commanded to the rotors, and inconsistencies in the predictions with respect to the real behaviour may allow to detect rotor failures.

This approach was studied in the literature. In [43, 44], a sliding mode observer is proposed to deal with partial or total failures in an octorotor, combined with an LPV or dynamic control allocation for a 4DOF, fault tolerant octorotor.

In [45], a nonlinear observer is proposed for a coaxial octorotor, where the fault detection is achieved by a deviation of the expected behaviour through residues analysis, and the isolation of the fault is obtained by observing the direction of rotation in the three axes after a failure occurs. An improvement of this work is shown in [46], where a nonlinear sliding mode observer is used for detection with the same isolation technique, and experimental results for up to four specific rotor failures are presented.

The work in [37, 38] proposes a bank of Luemberger observers, one for the nominal plant, and one for each of the possible failures considered, where the detection and isolation of a total rotor fault (impossibility to exert force) is achieved by analyzing the residues. It also establishes the conditions needed to find a common virtual actuator that enables the system to recover from failures that are non-isolable. Simulated and experimental results of the detection and isolation algorithm are shown for a coaxial octorotor and a PPNNPN hexarotor, respectively.

There are also works that approach the issue of fault detection and isolation through the dynamical model by a statistical approximation instead of an analytical model, such as the works in [47], where an algorithm is presented based in supervised learning using a random forest classifier and in [48], where a statistical time series is used for the same purpose. This kind of modelling requires a great amount of training data of the behaviour of the system, either from the real vehicle analyzed, or from an accurate simulated model.

4. Conclusion

Nowadays, given the proliferation of unmanned aerial vehicles that has been possible due to a reduction in production costs and an increasingly simple operation, fault tolerance has become a critical issue to ensure safety, both of the system and of third parties.

While covering several issues that have been researched in the last years, this chapter has focused mainly in a particular type of failure in multirotor vehicles, that where one of the rotors fails completely and is incapable of exerting thrust or torque. This is one of the most critical failures in this kind of systems, as it affects its maneuverability and flight time. Additionally, a definition of fault tolerance was proposed, stating that is of interest to maintain independent control in attitude and altitude (4DOF) in case of a failure.

It was stated that to achieve fault tolerance in a standard multirotor, a minimum of six rotors is needed. Considering this, a geometrical analysis over the force-torque matrix of an hexarotor is presented; by analyzing the matrix rank and its null space, it is possible to assess whether the vehicle is fault tolerant or not. However, this analysis by itself cannot evaluate the performance of the vehicle in case of a failure. This fact was proved for a symmetric, inwards-tilted hexarotor that, while practically capable of 4DOF control in case of a failure, has its maneuverability extremely limited, and therefore is unsuitable for real outdoor missions.

To conclude, a reconfigurable inward-tilted hexarotor design was presented, which relied on actively tilting two of its rotors in case of a failure in order to improve maneuverability. This vehicle proved to be capable of transitioning adequately between its nominal and failure states, given that the fault is detected in a reasonable time. Nevertheless, it has to be taken into account that the need of electromechanical devices to tilt the rotors (servomotors in the presented case) is an additional source of possible failures, therefore designs of this kind should be approached carefully to minimize the impact on the reliability of the system.

As small multirotor vehicles become ubiquitous, taking concrete actions to ensure safety is paramount. Complementing the positive steps being already taken by the industry –for example with the definition of no-fly zones– with other approaches such as the ones proposed in this chapter will enable a diverse variety of applications from which society can greatly benefit from.

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