Dispersive waves in optical fibers with a zero-nonlinearity wavelength

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Abstract: We present results on dispersive waves radiated by solitons in the context of fibers with arbitrary frequency-dependent nonlinearities. In particular, we focus on the effect of a zero-nonlinearity wavelength within the spectral region of interest.

Dispersive waves radiated by solitons play a fundamental role in supercontinuum generation (SCG). In particular, Cherenkov radiation [1] and radiation trapping [2] are two important phenomena involved in SCG that have been widely studied. However, there are few works in the literature on a complete analysis of these phenomena in the presence of a zero-nonlinearity wavelength (ZNW) [3]. In this work, we present the first steps in this direction. First, we show that new mathematical tools are needed, as the generalized nonlinear Schrödinger equation (GNLSE) [4], most frequently applied to model optical pulse propagation, fails to preserve the number of photons even in the case of lossless waveguides. We have recently introduced a new modeling equation, the photon-conserving generalized Schrödinger equation (pcGNLSE) [5], which circumvents this major problem. The pcGNLSE reads

$$\partial_z \tilde{A} = i\tilde{\beta}(\Omega)\tilde{A} + i\frac{\Gamma(\Omega)}{2}\mathscr{F}\left\{C^*B^2\right\} + i\frac{\Gamma^*(\Omega)}{2}\mathscr{F}\left\{B^*C^2\right\} + i f_\mathrm{R}\Gamma^*(\Omega)\mathscr{F}\left\{B\int_0^\infty h_\mathrm{R}(t')|B(t-t')|^2 dt' - B|B|^2\right\},$$

where $\tilde{A}$ is the Fourier transform of the pulse envelope A, $\tilde{\beta}(\omega)$ is the dispersion, f_R is the fractional Raman contribution, h_R is the impulse response corresponding to stimulated Raman scattering, and $\mathscr{F}\{\cdot\}$ denotes the Fourier transformation. B, C and Γ are defined by $\tilde{B}(\Omega) = \sqrt[4]{\gamma(\Omega)/(\omega_0+\Omega)}\tilde{A}(\Omega)$, $\tilde{C}(\Omega) = \left(\sqrt[4]{\gamma(\Omega)/(\omega_0+\Omega)}\right)^*\tilde{A}(\Omega)$, $\Gamma(\Omega) = \sqrt[4]{\gamma(\Omega)\cdot(\omega_0+\Omega)^3}$, where ω_0 is the central frequency, Ω is the detuning from ω_0, and $\gamma(\Omega)$ is the nonlinear coefficient.

Using the pcGNLSE, we study the phenomenon of radiation trapping in the presence of a ZNW. We show how a ZNW on the red side of the spectrum slows down the frequency shift of solitons due to intrapulse Raman scattering (IRS) and, concomitantly, the blue shift of trapped radiation. When a ZNW is located on the blue side of the spectrum the frequency shift due to IRS continues without restrain, as it also does the trapped radiation. We also study the influence of zero nonlinearities on Cherenkov radiation. In particular, we show that the GNLSE overestimates the amount of radiation when the ZNW approaches the red side of the soliton.

In order to study radiation trapping in the presence of a ZNW, we simulate the propagation of a 200-fs (FWHM) unchirped *sech* pulse at $\lambda_0 = 850$ nm with a peak power $P_0 = 6$ kW. The nonlinear coefficient at λ_0 is $\gamma_0 = 0.018\ \mathrm{W}^{-1}\,\mathrm{m}^{-1}$ and is varied linearly with frequency so that the ZNW ranges from 600 nm to 1000 nm. The dispersion at λ_0 is $\beta_2 = -23.9\ \mathrm{ps}^2\,\mathrm{km}^{-1}$ with a zero-dispersion wavelength at 780 nm. Except for the zeros of the nonlinear coefficient, these parameters are the same as those in Ref. [2]. Figure 1 shows results when ZNW= 600 nm (first two panels on the left) and ZNW= 1000 nm (rightmost panel). The plot in the first panel puts in evidence the problem of resorting to the GNLSE for modeling this case, as it presents an unphysical 12-fold increase in the number of photons resulting from the pulse redshift by IRS. Due to this inadequacy of the GNLSE, the remaining results correspond to those obtained with the pcGNLSE. The two panels on the right of Fig. 1 show spectrograms after 1-m propagation. The plot in the second panel shows the behavior of the trapped radiation when the ZNW lies in the blue side of the spectrum. As it can be observed, the ZNW does not inhibit radiation trapping and the trapped energy shifts towards the blue side as solitons, resulting from the fission of the input pulse, shift in the opposite direction. A different behavior is observed in the rightmost panel of Fig. 1

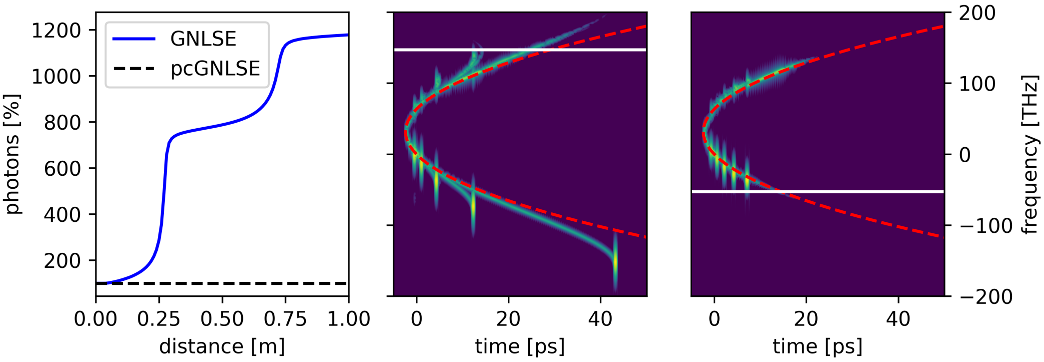

Fig. 1. Radiation trapping. Evolution of the number of photons according to the GNLSE and the pcGNLSE, and for ZNW = 600 nm (left). Output spectrograms for ZNW = 600 nm (middle) and 1000 nm (right). Solid white lines mark the position of the ZNW. Dashed red lines show the path of linear waves according to the dispersion relation. The center wavelength is at 850 nm.

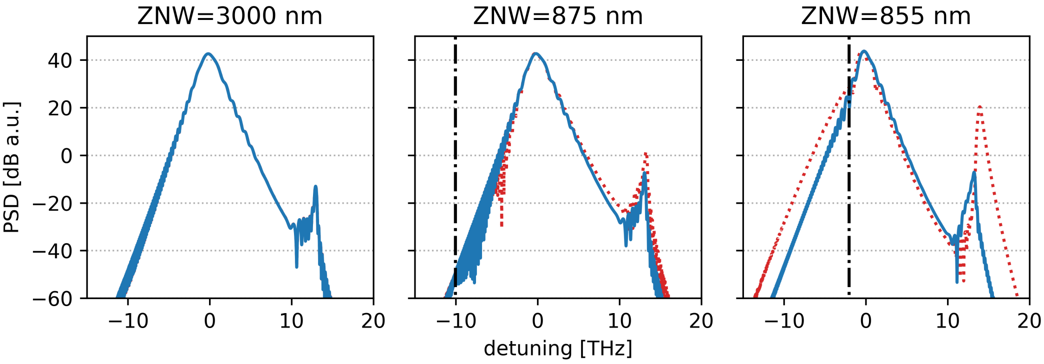

Fig. 2. Output spectra of a 30 m-long fiber for different ZNWs. The dotted red line corresponds to the GNLSE and the solid blue to line the pcGNLSE. The ZNW is marked with a dot-dashed black line as a reference.

where the ZNW lies in the red side of the spectrum. Pulse redshifting is restrained by the presence of the zero nonlinearity and pulses ‘bunch’ close to the ZNW. Although the blueshift of trapped radiation is also halted, part of the energy leaks towards longer wavelengths.

In order to study Cherenkov radiation, we launch a 200-fs fundamental soliton in a fiber with zero dispersion at 840 nm and the same γ_0 as before, and for different values of the ZNW. This kind of radiation is, in essence, produced by a phase-matched linear dispersive wave mechanism pumped by the soliton. Thus, we expect to observe noticeable differences whenever the ZNW affects soliton dynamics, as clearly seen in Fig. 2, where radiation corresponds to the spectral peak lying beyond 10 THz in all three cases. The GNLSE overestimates the amount of radiation whenever the ZNW approaches the red side of the soliton, while exhibiting an unphysical decrease in the number of photons.

References

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