A visual EEG epilepsy detection method based on a wavelet statistical representation and the Kullback-Leibler divergence

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Abstract— **This paper presents a statistical signal processing method for the characterization of EEG of patients suffering from epilepsy. A statistical model is proposed for the signals and the Kullback-Leibler divergence is used to study the differences between *Seizure/Non-Seizure* in patients suffering from epilepsy. Precisely, EEG signals are transformed into multivariate coefficients through multilevel 1D wavelet decomposition of different brain frequencies. The generalized Gaussian distribution (GGD) is shown to model precisely these coefficients. Patients are compared based on the analytical development of Kullback-Leibler divergence (KLD) of their corresponding GGD distributions. The method has been applied to a dataset of 18 epileptic signals of 9 patients. Results show a clear discrepancy between *Seizure/Non-Seizure* in epileptic signals, which helps in determining the onset of the seizure.**

Keywords— **Kullback-Leibler divergence, Epilepsy, Seizure/Non-Seizure, Multivariate wavelet decomposition, Generalized Gaussian distribution.**

I Introduction

The International League Against Epilepsy (ILAE) [1] defines an *epileptic seizure as a transient occurrence of signs and/or symptoms due to abnormal excessive or synchronous neuronal activity in the brain*. The different types of seizures depend on the location in the brain where it originated and on how far and fast it spreads. The correct identification of this onset location is key to a proper treatment. Electroencephalography (EEG) is a non-invasive and widely available biomedical modality that is used to diagnose epilepsy and plan treatment; neurologists trained in EEG are able to identify visually the onset and subsequent seizure through analysis of characteristic waveforms associated with seizures. This problem has been addressed in various research works such as [2–8], but remains an open issue. In this work, we adopt a statistical approach to distinguish *Seizure/Non-Seizure* in epileptic signals. The data is represented using the generalized Gaussian distribution in the wavelet domain. The analytical development of Kullback-Leibler Divergence (KLD) or relative entropy, is used to measure the discrepancy between probability density functions (PDF), specifically among the PDFs of the generalized Gaussian distributions for *Seizure/Non-Seizure* signals. See [9–11] for some works on this topic in epilepsy and [12–14] for some applications in EEG signals. The remainder of this paper is structured as follows. Section 2 presents the proposed methodology and details the generalized Gaussian model and the analytical development of Kullback-Leibler divergence. Section 3 describes the experimentation on real EEG signals and the presents results obtained. Section 4 discusses the findings and provides perspectives for future work.

II Methodology

The methodology used to analyze the EEG signals has three stages. The first stage is to represent the signals using a time-frequency Dauchebies wavelet decomposition [15, 16] with 6 scales, this gives the bands delta (0.5-4Hz), theta (4-8Hz), alpha (8-13Hz), beta (13-30Hz) and gamma (>30Hz). The aim of this stage is to assess the distribution of the energy throughout the frequency spectrum. The second stage consists in summarizing the information contained in each group (band and scale) of wavelet coefficients. The approach adopted consists in fitting the generalized Gaussian distribution statistical model to each group. The parameters α and β are estimated for each PDF using (3) giving a parameter vector that represents each group [17]. The third stage consists in measuring the difference between *Seizure/Non-Seizure* in epileptic signals by calculating the Kullback-Leibler Divergence (7) between generalized Gaussian distribution PDFs obtained for each patient.
We now introduce the generalized Gaussian distribution and Kullback-Leibler Divergence.

A Generalized Gaussian distribution

The univariate generalized Gaussian distribution (GGD) is a flexible statistical model for one-dimensional signals that

has found numerous applications in science and engineering. Its application to epilepsy signal has been studied in [18–20]. Since the wavelet detail coefficients arise from high-pass filtering a zero-mean EEG signal matrix, it can be safely assumed that they also have mean value of zero [21]. Consequently, the wavelet coefficients can be modeled through the parameters of the GGD [22] whose probability density function (PDF) is given by

$$f(x;\mu,\sigma,\beta) = \frac{\beta}{2\alpha(\sigma)\Gamma(\frac{1}{\beta})}\exp\left(-\frac{|(x-\mu)|^{\beta}}{2\sigma^2}\right) \quad (1)$$

$$\alpha(\sigma) = \sigma\sqrt{\frac{\Gamma(\frac{1}{\beta})}{\Gamma(\frac{3}{\beta})}}, \quad \Gamma(z) = \int_0^{+\infty} t^{z-1}e^{-t}\mathrm{d}t, z>0 \quad (2)$$

where $\mu \in \mathbb{R}$ is a location parameter, $\alpha \in \mathbb{R}^+$ is a scale parameter and $\beta \in \mathbb{R}^+$ is a shape parameter that controls the shape of the density tail. In the case of a zero-mean GGD, (1) can be written as

$$f_{\mathrm{GGD}}(x;\alpha,\beta) = \frac{\beta}{2\alpha\Gamma(\beta^{-1})}\exp\left(-\left|\frac{x}{\alpha}\right|^{\beta}\right) \quad (3)$$

where α replaces the scale parameter σ.

It should be noted that the GGD parametric distribution family includes many popular distributions that are commonly used in biomedical signal processing. For example, setting $\beta = 1$ leads to a Laplacian or double-exponential distribution, $\beta = 2$ leads to Gaussian or normal distribution, and $\beta \to \infty$ leads to a uniform distribution.

The GGD was fitted using a window shift of two seconds with overlapping of one second in 18 epileptic signals, for each signal obtained 205 fits on average for each epoch and calculated the parameters related to the scale (α) and shape (β) for each rhythm band. We refer the reader to [23] for a comprehensive treatment of the mathematical properties of the GGD model and [17,22] for a detailed explanation on the estimation of the GGD parameters.

B Kullback-Leibler Divergence

Let p and q two PDFs, then a Kullback-Leibler Divergence (KLD) [24] is given by

$$D_{KL}(p||q) = \int_{-\infty}^{\infty} log\left(\frac{p_x(x)}{q_x(x)}\right)p_x(x)dx \quad (4)$$

$$\begin{aligned} D_{KL}(p||q) &= -\int_{-\infty}^{\infty} log(q_x(x))p_x(x)dx + \\ &\int_{-\infty}^{\infty} log(p_x(x))p_x(x)dx = H_c(p,q) - H(x) \end{aligned} \quad (5)$$

Notice that in general $D_{KL}(p||q) \neq D_{KL}(q||p)$, and that $D_{KL}(p,q) = 0$ if and only if $p = q$ [25].
Rewriting the equation (3), the probability density function of GGD is given by

$$p(x,\alpha,\beta) = \frac{e^{-|\frac{x}{\alpha}|^{\beta}}}{2\alpha\Gamma[1+\beta^{-1}]} \quad (6)$$

Here we consider the divergence between two generalized Gaussian models with parameters (α_1,β_1,μ_1) and (α_2,β_2,μ_2) subject to the constraint $\mu_1 = \mu_2 = 0$ because our signals have zero mean. This divergence is given by

$$KLD_{pdf}(p||q) = \int_{-\infty}^{\infty} p(x,\alpha_1,\beta_1)log\left(\frac{p(x,\alpha_1,\beta_1)}{p(x,\alpha_2,\beta_2)}\right)dx \quad (7)$$

$$= \int_{-\infty}^{\infty} p(x,\alpha_1,\beta_1)\left[\left|\frac{x}{\alpha_2}\right|^{\beta_2} - \left|\frac{x}{\alpha_1}\right|^{\beta_1} + log\left(\frac{\alpha_2\Gamma(1+\frac{1}{\beta_2})}{\alpha_1\Gamma(1+\frac{1}{\beta_1})}\right)\right]dx$$

$$= \frac{-\Gamma(\frac{1}{\beta_1}) + (\frac{\alpha_1}{\alpha_2})^{\beta_2}\beta_1\Gamma(\frac{1+\beta_2}{\beta_1}) + \beta_1\Gamma(\frac{1}{\beta_1})log\left(\frac{\alpha_2\Gamma(1+\frac{1}{\beta_2})}{\alpha_1\Gamma(1+\frac{1}{\beta_1})}\right)}{\beta_1\Gamma(\frac{1}{\beta_1})}$$

$$KLD_{pdf}(p||q) = -\frac{1}{\beta_1} + \frac{(\frac{\alpha_1}{\alpha_2})^{\beta_2}\Gamma(\frac{1+\beta_2}{\beta_1})}{\Gamma(\frac{1}{\beta_1})} + log\left(\frac{\alpha_2\Gamma(1+\frac{1}{\beta_2})}{\alpha_1\Gamma(1+\frac{1}{\beta_1})}\right)$$

We compare the PDFs obtained 18 epileptic signals, using the scales and the shapes of the GGD using (6), in two stages in steps of one second without overlapping for each rhythms brain bands

1. Between sliding window and the seizure onset

$$KLD_{pdf}(p^{(i)}||q_{onset}) = \mathbb{W}^{(i)}KLD_{pdf}(p||q)$$

2. Between continuous PDFs coupled with a 7-order one-dimensional median filter [26]

$$KLD_{pdf}(p^{(i)}||q^{(i+1)}) = \mathbb{W}^{(i)}\mathbb{F}^{(i)}KLD_{pdf}(p||q)$$

with

$$\mathbb{W}^{(i)} = \left[0^{L\times iL}, I^{L\times L}, 0^{L\times N-iL-L}\right]$$
$$\mathbb{F}^{(i)} = medianFilter(KLD_{pdf}(p^{(i)}||q^{(i)}))$$

where $0^{N\times M} \in \mathbb{R}^{N\times M}$ is the null matrix, $I^{N\times N} \in \mathbb{R}^{N\times N}$ is the identity matrix and L is the number of measurement obtained in one second. We refer the reader to [25, 27–29] for a comprehensive treatment of the mathematical properties of the KLD statistical theory.

III Results and Discussion

The performance of the proposed statistical method was evaluated using the Children's Hospital Boston database [30, 31], which consists of 36 EEG recordings from pedi-

atric subjects with intractable seizures. In this work we used 18 seizures from 9 subjects. Datasets including two to five bipolar EEG recordings sampled at 256Hz were available for each subject. Each recording contained a seizure event with a labeled onset that was detected by an experienced neurologist, who worked backward from the observed clinical onset to find the epilepsy seizure onset. The signals used have one epoch focused on *Seizure/Non-Seizure* where the onset of the *Seizure* begins at two minutes.

We obtained a good performance of the KLD method via visual inspection in all 18 epileptic signals by an experienced neurologist. For illustration, Figures (1) and (2) depicts the different brain rhythms: delta, theta, alpha, beta and gamma, where the *Seizure* is 40 seconds of duration, we can see increase in activity between 2 minutes and 2.4 minutes in all brain rhythms. In Figure (1) we can notice how the signal have the *Seizure* onset begins at minute two; we can see clearly the discrepancy between *Seizure/Non-Seizure* in epileptic signals; while in Figure (2) the *Seizure* onset is detected clearly given by the highest peak which emerges from background of EEG showing a discrepancy between *Seizure/Non-Seizure*. Once the *Seizure* finished, there are several medical pathological factors that causes the signal takes a time in stabilize it again, this is the reason why the *Seizure* does not have an instantaneous change after the 2 minutes 40 seconds, however, is clear the discrepancy after the *Seizure*.

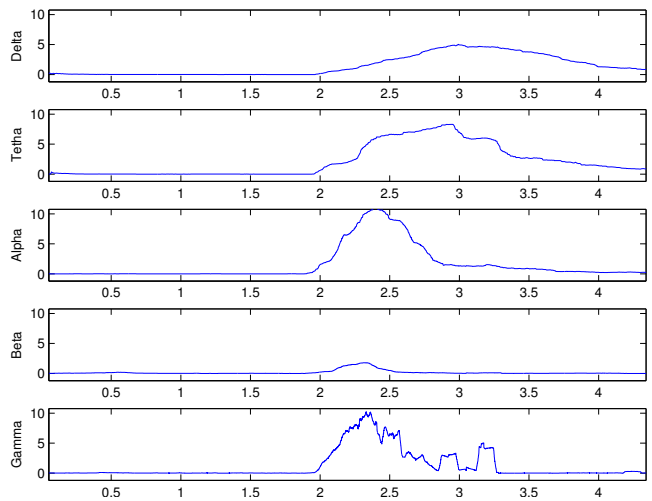

Fig. 1: KLD illustration between sliding window and the *Seizure* onset of the epileptic signal, showing a clear discrepancy between *Seizure/Non-Seizure*. In this example the *Seizure* onset begin at minute 2, and its duration is 40 seconds.

IV Conclusions

The preliminary results reported in this work in 18 epileptic signals suggest that the proposed method, is potentially useful for differentiating *Seizure/Non-Seizure* signals in epileptic signals and for onset detection.

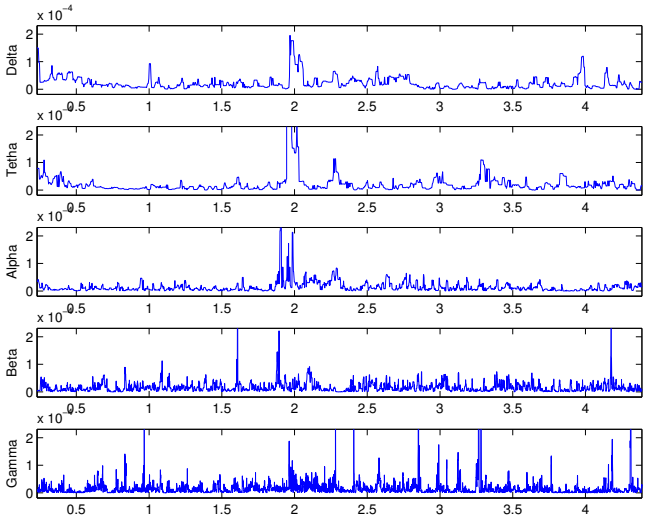

Fig. 2: KLD illustration between continuous PDFs coupled with a 7-order one-dimensional median filter, showing clearly the discrepancy given by the highest peak which emerges from background of EEG. In this example the *Seizure* onset begin at minute 2, and its duration is 40 seconds.

V Future Work

Perspective for future work include an extensive evaluation of the proposed methodology, as well as performing comparisons with other methods from the state of the art, and the development of fusion techniques to combine detections from several algorithms to increase robustness to noise and to artifacts.

It is also interesting to investigate how implement this method in real time for *Seizure/Non-Seizure* EEG classification using shift windows with different filters, optimizing display scales and calculating the onset delay by comparing the average amplitude with the background, similar to [19, 32].

Conflict of Interest

The authors declare that they have no conflict of interest.

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