TIME-DELAY PROPERTIES OF A STOCHASTIC-RESONANCE INFORMATION TRANSMISSION LINE

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In this paper we analyze the properties of a chain of forward-coupled bi-stable over-damped oscillators. It is well known that this system displays stochastic resonance and behaves as a transmission line when an adequate amount of noise is added to each oscillator, and the first oscillator is driven by a periodic (sine) modulating signal. By driving the first oscillator with a modulated sequence of random non-return-to-zero (NRZ) bits, we start by showing that the system exhibits a stochastic-resonance behavior. Then we show that bit delays can be adjusted by either changing the amount of noise and/or the coupling intensity between adjacent oscillators. We find that the system can be regarded as a tunable delay line for a broad range of noise and coupling parameters, a feature that may find applications in information processing and bit regeneration.

Keywords: Stochastic resonance; delay; transmission line.

1. Introduction

The stochastic-resonant behavior of a chain of bi-stable forward-coupled oscillators has previously been studied in a variety of contexts, including one-way driven dynamical chains [1] and synaptic transmission between neurons [2]. As in the general case of nonlinear systems exhibiting stochastic-resonance (SR) characteristics, the output signal-to-noise ratio is maximized for a given amount of input noise. Moreover, dynamic systems comprised of a chain of bi-stable double-well potentials driven by a periodic (sine) signal have been extensively studied and were shown to be able to sustain noise-assisted fault-tolerant transmission [2, 3].

In this paper we are interested in further exploring the transmission properties of these systems, focusing our attention on their time-delay characteristics. The problem of time synchronization aided by noise has been previously addressed in the context of dynamical systems [4, 5] and of stochastic resonance [6]. In this work we seek to answer whether it is possible to employ a chain of bi-stable oscillators as a 'tunable' delay line, an application often encountered in communication systems that has recently become very relevant in optical systems that employ phase coding instead of amplitude coding [7]. Also, this may find applications in the field of optical regeneration, as some schemes for optical pulse amplification and shaping (known as "2R regeneration") mimic fast saturable-absorbers [8], which are known to exhibit an SR behavior [9].

The system under consideration consists of a chain of (arbitrarily) ten bi-stable double-well forward-coupled potentials such as the ones described in [1–3]. The equation of motion of the n-th oscillator is described by the following stochastic differential equation

$$dX_n = \left(-\frac{\partial}{\partial x} U(X_n) + \varepsilon X_{n-1} \right) dt + \sigma\eta(t)\, dt \,, \tag{1}$$

where $U(x)$ is the potential defined by

$$U(x) = U_0 \left(\frac{x}{x_c} \right)^2 \left[\left(\frac{x}{x_c} \right)^2 - 2 \right], \tag{2}$$

ε is the coupling-strength between adjacent oscillators, εX_0 is the driving force of the first oscillator, σ^2 is the noise intensity and $\eta(t)$ represents white Gaussian noise. It is usually convenient to re-write Eq. (1) as

$$dX_n = -\frac{\partial}{\partial x} \tilde{U}_n(X_n) dt + \sigma\eta(t)\, dt \,, \tag{3}$$

where the potential $\tilde{U}_n(x)$ is given by

$$\tilde{U}_n(x) = U(x) - \left(\varepsilon X_{n-1} \right) x. \tag{4}$$

We drive the first oscillator in the chain with a pseudo-random sequence of bits coded in non-return-to-zero (NRZ) format [10], a coding format that is used in most high-speed transmission applications, especially in fiber-optic communication systems. It is interesting to note that the system under consideration operates in the nonlinear regime of SR [11], as we are interested in transmitting bits that carry a limited amount of noise.
The input sequence is thus described by

$$\varepsilon X_0(t) = F_0 \sum_{j=0}^{N-1} B_j S(t - jT_B) \,, \tag{5}$$

where F_0 is the drive intensity, T_B is the bit slot, B_j is randomly chosen from {-1,+1}, and $S(t) = 1$ for $0 < t < T_B$ and $S(t) = 0$ elsewhere.

We solve Eq. (1) by means of the Euler-Maruyama method (see, e.g., [12]) and choose simulation parameters in agreement with those of [1] namely $U_0 = 256$, $x_c = \sqrt{32}$, $F_0 = 40$ and $T_B = 5$.

In our simulations input sequences consist of 2048 bits. Mean bit delays are estimated by averaging cross-correlation results between the input and output bit streams, obtained from 200 different realizations of noise, input bits, and randomly-chosen initial states for each oscillator. As previously mentioned, we are interested in investigating whether it is possible to 'tune' these delays either by changing the amount of noise fed into each and every oscillator and/or by changing the coupling strength between adjacent oscillators.

Now we turn to the computation of SNRs for the case of a modulated driving signal. First, we calculate the signal power by integrating its power spectrum up to twice the signal rate (corresponding to the spectrum's second minimum)

$$P_{\text{signal}} = \int_0^{2/T_B} \left| x(\nu) \right|^2 \, \mathrm{d}\nu \, . \tag{6}$$

Noise power is estimated by integrating the signal a narrow bandwidth around the signal's first minimum

$$P_{\text{noise}} = \frac{\int_{1/T_B-\Delta}^{1/T_B+\Delta} \left| x(\nu) \right|^2 \, \mathrm{d}\nu}{\Delta \, T_B} \, , \tag{7}$$

where Δ is chosen as ten times the simulated frequency step-size.

We then subtract P_{noise} from P_{signal} and calculate the SNR in dBs

$$\text{SNR}\big|_{\text{dB}} = 10 \, \log\left(\frac{P_{\text{signal}}}{P_{\text{noise}}} - 1 \right) \tag{8}$$

In Fig. 1 we explicitly show that the oscillator chain exhibits an SR behavior by plotting the obtained output Signal-to-Noise Ratio (SNR) as a function of noise intensity. By comparing these results with those of Mc Namara *et al.* [1] we find that the SNR peak is shifted towards lower noise levels. This can be understood by noting that in Ref. [1]

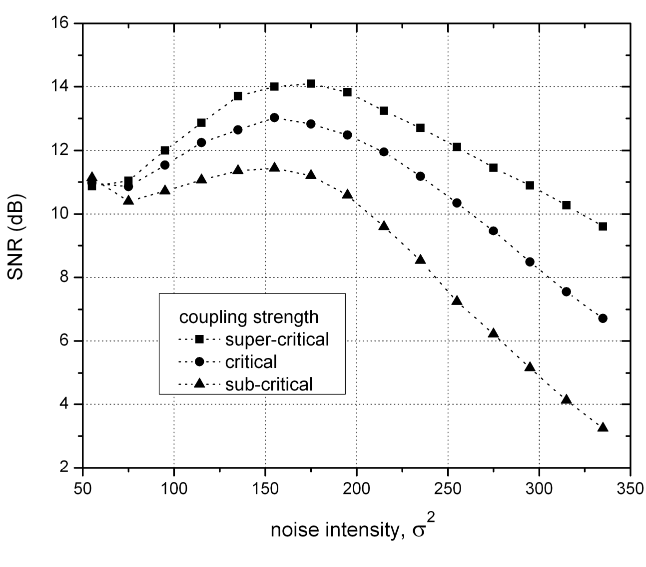

Fig. 1. Output SNR (after the 10th oscillator) as a function of noise and for different coupling strengths. Simulation parameters are $U_o = 256$, $x_c = 5.66$, $F_0 = 40$, $T_B = 5$, N = 2048 bits.

the driving force is a harmonic signal; in this way the potential in (4) remains tilted for a shorter time as compared to the case of square NRZ bits, thus requiring a larger noise contribution in order for the oscillator to make a transition.

Results in this paper are shown for three different coupling strengths, which we call 'super-critical', 'critical', and 'sub-critical'. A 'super-critical' coupling strength is strong enough to guarantee operation of the transmission line without the addition of noise, i.e. the line is no longer 'SR-driven'. In the 'sub-critical' case, addition of noise at each oscillator is required in order to sustain transmission. The critical coupling strength can be estimated by searching for the minimum coupling that no longer yields two distinct stable minima in the potential in (4), and is given by [13]

$$\varepsilon_c \approx 1.54 \frac{U_0}{x_c^2}, \tag{9}$$

which in our case corresponds to $\varepsilon_c \sim 12.5$.

From results in Fig. 1 we identify a relevant noise range that maximizes the output SNR, in our case centered around a noise intensity of $\sigma^2 \sim 150$, and then explore this noise range for subsequent simulations.

2. Results and Discussion

In Fig. 2 we show bit delays (normalized to a bit slot) as a function of the added noise. We observe that a strong coupling leads to a weak dependence of the bit delay with noise and, vice versa, a sub-critical coupling leads to a much broader tuning range, in our case up to 60% of the bit slot after ten oscillators.

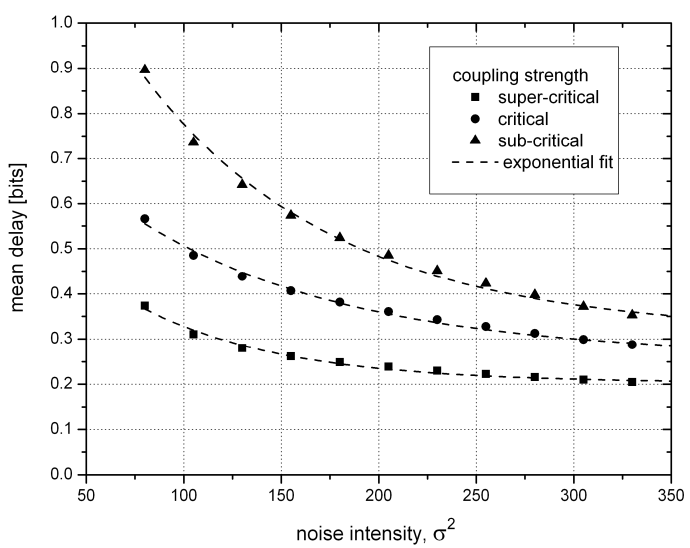

Fig. 2. Mean bit-delays (normalized to a bit slot) as a function of noise for super-, sub-, and critical coupling strengths. Delays decay exponentially with noise. Simulation parameters are those of Fig. 1. Mean delays are calculated by averaging 200 noise realizations.

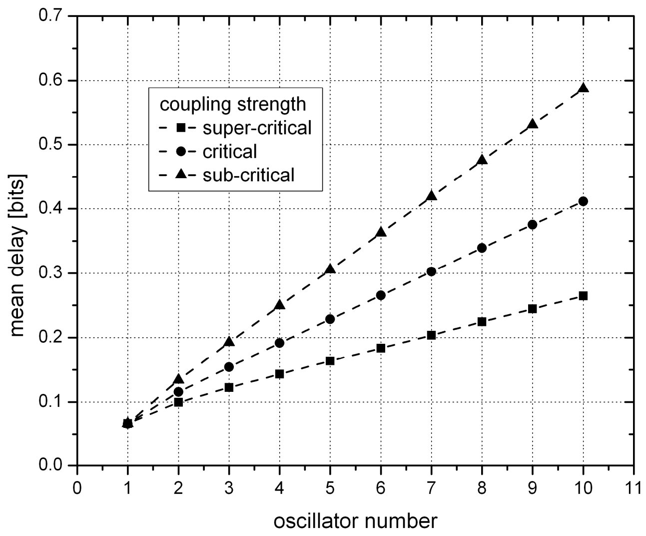

Fig. 3. Mean bit-delays as a function of oscillator number and for different coupling strengths. Simulation parameters are those of Fig. 1.

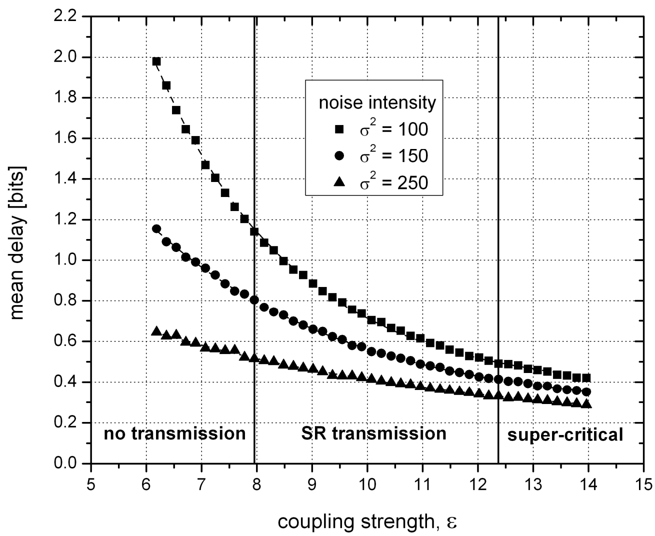

Fig. 4. Mean bit-delays (normalized to a bit slot) as a function of coupling strength and for different amounts of noise. Three regimes are observed: no transmission, SR driven transmission, and super-critical (not driven by noise) transmission. Simulation parameters are those of Fig. 1.

It is important to note that there are two physical limits for the added noise and for a sub-critical coupling: the low end of the range corresponds to an amount of noise that is not sufficient to sustain transmission; on the other hand, at the higher end of the range, the degraded output SNR renders the transmitted bits unrecognizable.

In practical terms, one is usually interested in delays of the order of one bit slot, an amount that allows, for instance, retrieving phase-encoded signals [4]. Our results indicate that this is easily attainable for the chosen system parameters and for a slightly longer line, as shown in Fig. 3, where bit delays are found to increase linearly with the number of oscillators.

Next, we investigate the time-delay dependence on the coupling strength of adjacent oscillators. Results are shown in Fig. 4 where we observe an exponential decay with growing coupling strengths. In Fig. 4 we identify different regimes: for small coupling strengths the line is not able to start transmission. On the other hand, for large coupling-strengths the line sustains transmission even in the absence of noise. The range in between corresponds to a truly 'SR-driven', i.e. noise- supported, transmission line. Observe that, in all cases, larger bit delays are obtained for less added noise.

The exponential trends observed in Figs. 2 and 4 can be understood in the following manner. The potential in Eq. (3) changes whenever the driving force of the n-th oscillator transitions between the low and high states, roughly corresponding to the potential minima $\pm x_c$ in Eq. (2). For the sake of simplicity, let us assume that a low-to-high transition occurs at time t_0, i.e., $X_{n-1}\left(t_0^-\right) \sim -x_c$ and $X_{n-1}\left(t_0^+\right) \sim +x_c$. Furthermore, let us assume that $X_n\left(t_0^-\right) \sim -x_c$. Then, the average delay of the n-th oscillator is given by the mean escape-time formula [1]

$$\bar{\tau} = \frac{\pi}{\sqrt{-\tilde{U}_n''(x_0)\tilde{U}_n''(x_1)}} \exp\left\{ \frac{\left[\tilde{U}_1(x_0) - \tilde{U}_1(x_1)\right]}{\sigma^2/2} \right\},$$

where x_0 and x_1 correspond to the maximum and (current) minimum of $\tilde{U}_n(x)$. If we assume that x_0~0, x_1~-x_c, and using the fact that $\tilde{U}_n''(x) = U_n''(x)$, we obtain

$$\bar{\tau} = \frac{\pi}{U_0} \exp\left\{ \frac{\left[\tilde{U}_n(0) - \tilde{U}_n(-x_c)\right]}{\sigma^2/2} \right\} = \frac{\pi}{U_0} \exp\left\{ \frac{2\left[U_0 - \varepsilon x_c^2\right]}{\sigma^2} \right\}. \tag{10}$$

Note that Eq. (10) holds within the approximation $\varepsilon x_c^2 << U_0$. Although the coupling-strength range we explore in this paper does not fulfill this condition, we can still observe a trend, as shown in Figs. 2 and 4, where bit delays decrease exponentially with increasing coupling strengths, and decrease with increasing noise intensity.

3. Conclusion

We analyzed the time-delay properties of a double-well forward-coupled SR-driven information transmission line, and showed that it can be regarded as a tunable delay line for a broad range of noise and coupling-strength parameters. We identified two coupling-strength regimes, corresponding to noise-supported and coupling-supported transmission, and for the former, discussed its practical range of validity.

Future work on this area includes the analysis of the dependence of the delays on the spectral content of the transmitted signal, i.e. the dispersion characteristics of the system, the study of the output signal integrity, and the possible application of these results to all-optical pulse regeneration.

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