

Vision-based Integrated Navigation System and Optimal Allocation in Formation Flying

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Abstract—This article proposes an integrated navigation system for multiple micro aerial vehicles flying in formation. A data fusion algorithm uses measurements from an inertial measurement unit, a GPS receiver, and a camera allowing to use the positioning information of the surrounding vehicles to improve its estimation. A measure of the navigation performance of the formation is defined. Based on such measure, the position where each vehicle should be located in the formation is studied to guarantee the best overall navigation quality.

I. INTRODUCTION

Several research projects with the goal of developing swarm of micro aerial vehicles (MAVs) have emerged in the last years. The reason is the broad spectrum of applications in which this kind of systems could be applied, e.g., rescue, surveillance, inspection of hazardous environments, among others. For instance, in [1], [2] a swarm of vision-controlled MAVs capable of autonomous navigation is presented. The swarm is used for 3D-mapping of the environment, where each vehicle collects data which is then fused and processed off-line for precise 3D-mapping. During the mission, vehicles are monitored and commanded with a ground station. Sharing navigation information among MAVs can improve the performance of the swarm, but additional computations and technological challenges must be taken into account, for instance inter-vehicle communication. Depending on the swarm mission, MAV hardware, and navigation sensors, the decision of sharing navigation information should be taken. This fact results in the need of a quantitative measure of how to improve the navigation performance of the system if navigation information is shared among vehicles.

The navigation system is a fundamental component for vehicle control, since it computes the navigation parameters (position, velocity and attitude) of the vehicle. Different sensors can be used to estimate the navigation parameters. For instance, inertial navigation systems compute navigation parameters numerically integrating data provided by an inertial measurement unit (IMU), which contains three orthogonal gyroscopes and three orthogonal accelerometers. These systems are fundamental in several applications because they collect and provide autonomous information at high rates. Nevertheless, accumulated errors grow unbounded over time. To achieve an acceptable level of accuracy, a high quality IMU is required, which results in high cost and elevated size

and weight, making its use impractical for many applications like MAVs.

In contrast, navigation systems that depend on external signals (e.g. GPS, magnetometers, cameras) provide measurements with bounded errors but at slower data rates, which translates into limitations for certain applications. Moreover, these sensors are vulnerable to jamming or temporal occlusions of signals which affect their measurements.

The core of an integrated navigation system is a data fusion algorithm (usually an extended Kalman filter - EKF) that combines the on-board sensor information. This algorithm estimates the navigation parameters with bounded errors, high accuracy and high data rates. By doing this, the quality and cost of the inertial sensors can be drastically reduced. Fusing navigation information brings the ability of using small size and low weight inertial sensors in MAVs while still having a good navigation performance.

In this article we present an integrated navigation system to be used in MAVs flying in formation. The fusion algorithm incorporates measurements from an IMU, a GPS receiver, and a camera that allows to visually detect the surrounding vehicles. Integration of IMU and GPS data is common in the literature, see for example [3] and references therein. Different methodologies for complementing GPS/INS integrated navigation systems with vision sensors have been studied. For instance, Hoshizaki et al. [4] and Veth et al. [5] studied how the inclusion of feature-tracking methods improve the performance of tightly-coupled INS/GPS navigation system. In [6] Fakhri et al. proposed an adaptive method for merging information provided by a camera with inertial sensors and GPS. Priot et al. [7] proposed a method for performing inertial sensors calibration, which merged information of a deeply-coupled INS/GPS navigation system and images provided by a camera.

Using a camera as an exteroceptive sensor not only complements GPS/INS systems, but it can also help maintaining a good estimate on GPS occlusion, as seen for example in [8]–[10]. In [8], [9], cameras are used on-board the vehicles to extract features from the environment and estimate UAVs poses. Saska et al. [10] use on-board cameras to detect fiducial markers placed on other vehicles to estimate relative distance, which was fused with IMU and Optical Flow measurements. In [11], a monocular camera is used to improve agents positioning, with a UAV flying over a group of unmanned ground vehicles (UGV) without state information sharing. Martinelli et al. [12] studied the integration of visual information from a monocular camera with that of an IMU in cooperative localization, sharing

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sensor data.

In this work we propose a simple loosely-coupled INS/GPS integration. Although techniques mentioned above may have better performance, the main contribution of this work is not the INS/GPS integration. Instead, we are interested in using camera measurements to obtain relative positioning with respect to other vehicles in the swarm/formation. These measurements allow sharing navigation information among vehicles, which can improve the navigation performance. In some sense, this is similar to the visual SLAM problem, where we replace landmarks with the surrounding vehicles.

The works mentioned so far have in common that all vehicles in the same domain are equal. This assumption is usually true, but not always holds. One can imagine a group of UGV with different sensors, as individual tasks may differ to achieve a global goal. This is easier to see in the case of MAVs, as their payload weight is a fundamental constraint.

Now, suppose that a formation of MAVs (with different quality of navigation systems) is performing a given task, also assume that vehicles have relative position information (for instance given by the camera). We are interested in studying the following problem: Given a fixed vehicle formation, is the navigation performance affected if we rearrange the order of the vehicles? For answering this question we study the navigation performance of the formation and analyze how this performance changes under vehicles permutations. Under certain assumptions we define the optimal vehicles arrangement.

Results presented here can be extended to other navigation sensors measuring relative positioning. The reason for using a camera to provide inter-vehicle relative information is mainly practical, since it is a small, light, and low cost sensor.

The outline of this paper is as follows: in section 2 we present notation and the preliminaries. In section 3 we introduce integrated navigation algorithms, focusing on the dynamics that describes the error behavior of these systems, which is crucial for implementing the integrated navigation algorithm. Section 4 is devoted to introducing a simplified model for the camera used as navigation sensor. In section 5 we present the integrated navigation algorithm that fuses inertial, GPS, and camera information. In section 6 we study the optimal formation problem exposed above and finally in section 7 we present simulation results.

II. PRELIMINARIES

In what follows, \mathbb{R}^n denotes the space of (column) vectors with n real components. Given a matrix $A \in \mathbb{R}^{n \times m}$, $A^T \in \mathbb{R}^{m \times n}$ denotes its transpose. $SO(3)$ denotes the special orthogonal group of matrices in $\mathbb{R}^{3 \times 3}$, i.e., orthogonal matrices with determinant equal to 1 (which is the set of matrices representing rotations in \mathbb{R}^3).

The operator $S : \mathbb{R}^3 \rightarrow \mathbb{R}^{3 \times 3}$ is defined as the (invertible) function that assigns to $x \in \mathbb{R}^3$, the unique skew-symmetric matrix $S(x) \in \mathbb{R}^{3 \times 3}$ such that $S(x)y = x \times y$, for every $y \in \mathbb{R}^3$, where \times denotes the standard cross product. In

fact, if $x = (x_1, x_2, x_3)^T$ then

$$S(x) = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}.$$

Given k matrices in $\mathbb{R}^{n \times m}$, we define the $\mathbb{R}^{kn \times km}$ matrix

$$\text{diag} \begin{pmatrix} A_1 \\ A_2 \\ \vdots \\ A_k \end{pmatrix} = \begin{bmatrix} A_1 & 0 & \dots & 0 \\ 0 & A_2 & \dots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \dots & A_k \end{bmatrix}$$

III. INERTIAL NAVIGATION SYSTEM

To determine the position, velocity, and orientation of a vehicle from inertial measurements (accelerometers and gyroscopes), the differential kinematic equations that relate such parameters must be integrated in real time [3]. This task is performed by the inertial navigation system (INS). To perform this operation, it is required to know the initial position, velocity, and attitude of the vehicle. Given that, this computation is based on the numerical integration of the kinematic equations. The accurate knowledge of the initial conditions, as well as the accuracy of the inertial instruments, are crucial for the navigation. Otherwise the errors could grow unboundedly with time. This is the main drawback of the INS, and it is not negligible in low cost inertial instruments, in particular those based on MEMS technologies [13].

A. Inertial navigation system error dynamics

In this section, an INS error model for each vehicle in the formation of MAVs is presented. This model is of great importance in the design of the integrated navigation system.

Let $p_i, v_i : \mathbb{R} \rightarrow \mathbb{R}^3$ and $C_i : \mathbb{R} \rightarrow SO(3)$ functions of time $t \in \mathbb{R}$ representing the (true) position, velocity and attitude of vehicle i , respectively. Let $\hat{p}_i, \hat{v}_i : \mathbb{R} \rightarrow \mathbb{R}^3$ and $\hat{C}_i : \mathbb{R} \rightarrow SO(3)$ the position, velocity and attitude of vehicle i computed by the INS.

As usual, the INS position and velocity errors are defined as

$$\delta p_i = \hat{p}_i - p_i, \quad (1)$$

$$\delta v_i = \hat{v}_i - v_i. \quad (2)$$

and the attitude error $\phi : \mathbb{R} \rightarrow \mathbb{R}^3$ is defined as

$$\phi_i = S^{-1}(\hat{C}_i C_i^T - I). \quad (3)$$

Remark III.1. It is well known that, a non-zero vector $\psi \in \mathbb{R}^3$ can be identified with the (unique) rotation matrix having $\psi \in \mathbb{R}^3$ as invariant vector and rotating any $v \in \mathbb{R}^3$, $\|\psi\|$ radians (defining a positive direction of rotation) around $\psi/\|\psi\|$. Moreover, this identification is given by

$$M(\psi) = I + \frac{\sin \|\psi\|}{\|\psi\|} S(\psi) + \frac{1 - \cos \|\psi\|}{\|\psi\|^2} S^2(\psi) \in SO(3).$$

Note that, for ψ small enough (neglecting second order terms) $M(\psi) \in SO(3)$ can be approximated by $\tilde{M}(\psi) =$

$I + S(\psi) \in \mathbb{R}^{3 \times 3}$. Then, for a given time $t \in \mathbb{R}$, the attitude error has an interesting physical interpretation. Since $\hat{C}(t) = (I + S(\phi(t)))C(t)$, for small $\phi(t) \in \mathbb{R}^3$, the matrix $\hat{M}(\phi(t)) = I + S(\phi(t))$ approximates the rotation matrix that compensates the cosine matrix calculated by the INS.

Close to the equilibrium point $(\delta p_i^T, \delta v_i^T, \phi_i^T) = 0 \in \mathbb{R}^9$ the INS error dynamics can be modeled as [3],

$$\dot{X}_i(t) = \tilde{A}_i(t)X_i(t) + \tilde{B}_i(t)\tilde{\xi}_i(t) \quad (4)$$

where,

$$\begin{aligned} X_i(t) &= (\delta p_i(t)^T, \delta v_i(t)^T, \phi_i(t)^T)^T, \\ \tilde{A}_i(t) &= \begin{pmatrix} 0 & I & 0 \\ \frac{g}{|\hat{p}_i(t)|} (3 \frac{\hat{p}_i(t) \hat{p}_i(t)^T}{|\hat{p}_i(t)|^2} - I) & 0 & -S(\hat{C}_i(t)f_i(t)) \\ 0 & 0 & 0 \end{pmatrix} \in \mathbb{R}^{9 \times 9}, \\ \tilde{B}_i(t) &= \begin{pmatrix} 0 & 0 \\ 0 & \hat{C}_i(t) \\ \hat{C}_i(t) & 0 \end{pmatrix} \in \mathbb{R}^{9 \times 6} \text{ and} \\ \tilde{\xi}_i(t) &= (\delta \omega_i(t)^T, \delta f_i(t)^T)^T \in \mathbb{R}^6. \end{aligned}$$

The specific force f_i and the angular velocity ω_i are the IMU measurements of vehicle i , which are corrupted with perturbations δf_i and $\delta \omega_i$, respectively. The gravity g is assumed a constant with value $9.798 \frac{m}{s^2}$.

Given the sequence of times $\{t_k\}_{k \in \mathbb{N}}$, it is possible to discretize equation (4) to obtain a discrete time model [3]:

$$X_i(t_{k+1}) = A_i(t_k)X_i(t_k) + B_i(t_k)\xi_i(t_k). \quad (5)$$

To design an integrated navigation algorithm, the IMU measurements perturbations are modeled as gaussian zero-mean stochastic processes $\{\xi_i(t_k)\}_{k \in \mathbb{N}}$ with realizations in \mathbb{R}^6 and covariance matrices $Q_i(t_k) = E(\xi_i(t_k)\xi_i(t_k)^T) > 0$. Thus, the INS error is the stochastic process $\{X_i(t_k)\}_{k \in \mathbb{N}}$, with realizations in \mathbb{R}^9 , given by equation (5).

If we have $n \in \mathbb{N}$ MAVs, then we can define the stochastic process $\{X(t_k)\}_{k \in \mathbb{N}}$, with realizations in \mathbb{R}^{9n} , given by $X^T(t_k) = [X_1^T(t_k), X_2^T(t_k), \dots, X_n^T(t_k)]$, $k \in \mathbb{N}$, which models the INS error of all the vehicles in the formation. Then, from equation (5) it follows that

$$X(t_{k+1}) = A(t_k)X(t_k) + B(t_k)\xi(t_k), \quad (6)$$

where

$$\begin{aligned} A(t_k) &= \text{diag} \begin{pmatrix} A_1(t_k) \\ A_2(t_k) \\ \vdots \\ A_n(t_k) \end{pmatrix}, \\ B(t_k) &= \text{diag} \begin{pmatrix} B_1(t_k) \\ B_2(t_k) \\ \vdots \\ B_n(t_k) \end{pmatrix} \text{ and } \xi(t_k) = \begin{bmatrix} \xi_1(t_k) \\ \xi_2(t_k) \\ \vdots \\ \xi_n(t_k) \end{bmatrix}. \end{aligned}$$

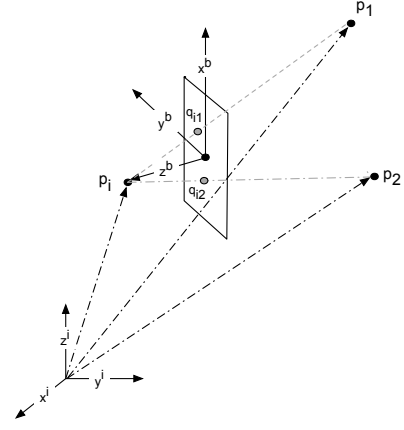


Fig. 1. Camera model

IV. CAMERA MEASUREMENTS MODEL

Suppose that we have a formation of n MAV, with positions $\{p_1(t), p_2(t), \dots, p_n(t)\}$. The purpose of this section is to present the method implemented to determine the position of vehicle i , by observing with a camera $k_i(t) \in \mathbb{N}$ vehicles, with known positions $p_j(t) \in \mathbb{R}^3$, $j \in \mathcal{I}_i(t) \subseteq \{1, 2, \dots, n\}$, $j \neq i$. Observe that, the number of vehicles in the field of view is not the same for each vehicle (this is the reason for subindex i in k_i) and it is a function of time. For example, suppose that we have 5 UAV flying in formation and, in time $t \in \mathbb{R}$, vehicle 1 can see vehicles $\mathcal{I}_1(t) = \{2, 4, 5\}$, ($k_1(t) = 3$).

In order to simplify notation, in what follows we will omit the dependence on time t . For defining the camera model, we will make reference to Figure 1. For a given time $t \in \mathbb{R}$, let $p_i = p_i(t) \in \mathbb{R}^3$ be the camera focal point of vehicle i , with focal distance d_f , and let π_f be the camera plane defined as

$$\pi_f = \{p \in \mathbb{R}^3 : (p - p_i + d_f z^b)^T z^b = 0\}.$$

Where (x^b, y^b, z^b) is an orthogonal frame with z^b normal to the plane π_f and x^b, y^b defining the axis of the image where the points q_{ij} are measured by the camera.

Notice that, since the camera is fixed to the vehicle, the frame (x^b, y^b, z^b) defines the attitude of the vehicle¹. Let $\alpha_{ij}, \beta_{ij} \in \mathbb{R}$ be the coordinates of point q_{ij} in the camera plane (i.e., the coordinates measured by the camera). If $C_i \in SO(3)$ is a matrix containing vectors x^b, y^b, z^b as columns (i.e., the matrix representing the attitude of vehicle i), then:

$$\begin{aligned} (q_{ij} - p_i)(p_j - p_i)^T \hat{z}^b &= C_i \begin{bmatrix} \alpha_{ij} \\ \beta_{ij} \\ -d_f \end{bmatrix} (p_j - p_i)^T \hat{z}^b = \\ &= -d_f (p_j - p_i), \end{aligned} \quad (7)$$

or more compactly, since $\hat{z}^b = C_i [0 \ 0 \ 1]^T$,

$$\tilde{M}_{ij} C_i^T (p_j - p_i) = 0, \quad (8)$$

¹This is the reason why the supraindex b is used, the *body* frame, as it is usually called.

where

$$\tilde{M}_{ij} = \begin{bmatrix} d_f & 0 & \alpha_{ij} \\ 0 & d_f & \beta_{ij} \end{bmatrix}.$$

This is a simplified standard camera model, see for instance [4], [14].

Remark IV.1. Notice that, to determine the position $p_i \in \mathbb{R}^3$ using only camera information it is necessary to know the attitude of vehicle i (i.e., $C_i \in SO(3)$) and the position of at least two vehicles $p_j, p_k, i \neq j, k$ satisfying, $\alpha_{ij} \neq \alpha_{ik}$ or $\beta_{ij} \neq \beta_{ik}$, i.e., two vehicles distinguishable with the camera. Indeed, suppose that $p_j, p_k, i \neq j, k$ then we can write

$$\begin{bmatrix} \tilde{M}_{ij} \\ \tilde{M}_{ik} \end{bmatrix} C_i^T p_i = \begin{bmatrix} \tilde{M}_{ij} C_i^T p_j \\ \tilde{M}_{ik} C_i^T p_k \end{bmatrix}. \quad (9)$$

If $\alpha_{ij} \neq \alpha_{ik}$ or $\beta_{ij} \neq \beta_{ik}$, it follows that

$$\begin{bmatrix} \tilde{M}_{ij} \\ \tilde{M}_{ik} \end{bmatrix} = \begin{bmatrix} d_f & 0 & \alpha_{ij} \\ 0 & d_f & \beta_{ij} \\ d_f & 0 & \alpha_{ik} \\ 0 & d_f & \beta_{ik} \end{bmatrix}$$

has full rank, then equation (9) has a unique solution.

If k_i (different) vehicles $\{p_{j_r}\}_{r=1,\dots,k_i}, j_r \in \mathcal{I}_i$ are in the field of view of the camera of vehicle i , and two of these vehicles satisfy condition given in Remark IV.1, position of vehicle i can be obtained by solving equation

$$\begin{bmatrix} \tilde{M}_{ij_1} \\ \vdots \\ \tilde{M}_{ij_{k_i}} \end{bmatrix} C_i^T p_i^c = \begin{bmatrix} \tilde{M}_{ij_1} C_i^T p_{j_1} \\ \vdots \\ \tilde{M}_{ij_{k_i}} C_i^T p_{j_{k_i}} \end{bmatrix}. \quad (10)$$

The supra-index in p_i^c is to remark that this position is obtained with measurements provided by the camera.

Let $\chi_{\mathcal{I}_i}(j) = \begin{cases} 1 & \text{if } j \in \mathcal{I}_i \\ 0 & \text{if } j \notin \mathcal{I}_i \end{cases}$ and $M_{ij} = \chi_{\mathcal{I}_i}(j) \tilde{M}_{ij}$. Notice that, equation (10) can be written as,

$$\begin{bmatrix} M_{i1} \\ \vdots \\ M_{in} \end{bmatrix} C_i^T p_i^c = \begin{bmatrix} M_{i1} C_i^T p_1 \\ \vdots \\ M_{in} C_i^T p_n \end{bmatrix}. \quad (11)$$

Then,

$$p_i^c = C_i \begin{bmatrix} M_{i1} \\ \vdots \\ M_{in} \end{bmatrix}^\dagger \begin{bmatrix} M_{i1} C_i^T p_1 \\ \vdots \\ M_{in} C_i^T p_n \end{bmatrix} = \left(\sum_{j=1}^n M_{ij}^T M_{ij} \right)^{-1} \left(\sum_{j=1}^n M_{ij}^T M_{ij} C_i^T p_j \right)$$

The matrix C_i needed to solve equation (10) can be obtained from the navigation system of vehicle i . The positions p_j of the other vehicles are provided by their respective navigation systems. As a result, inter-vehicle communication is required to share such navigation information.

A. Camera error model

The camera error model is obtained by a perturbation of equation (11), where the focal length d_f is assumed known. By Remark III.1, $\delta C_i = \hat{C}_i - C_i \approx S(\phi_i) C_i$. Then,

$$\begin{bmatrix} M_{i1} \\ \vdots \\ M_{in} \end{bmatrix} C_i^T \delta p_i^c = \begin{bmatrix} M_{i1} C_i^T S(p_1 - p_i) \\ \vdots \\ M_{in} C_i^T S(p_n - p_i) \end{bmatrix} \phi_i + \quad (12)$$

$$\begin{bmatrix} M_{i1} C_i^T \delta p_1 \\ \vdots \\ M_{in} C_i^T \delta p_n \end{bmatrix} + \begin{bmatrix} \delta M_{i1} C_i^T (p_1 - p_i) \\ \vdots \\ \delta M_{in} C_i^T (p_n - p_i) \end{bmatrix},$$

$$\text{where } \delta M_{ij} = \chi_{\mathcal{I}_i}(j) \begin{bmatrix} 0 & 0 & \delta \alpha_{ij} \\ 0 & 0 & \delta \beta_{ij} \end{bmatrix}.$$

Then,

$$-\delta p_i^c(t) = \Sigma_i \phi_i + \Delta_i \begin{bmatrix} \delta p_1 \\ \vdots \\ \delta p_n \end{bmatrix} + \Theta_i \begin{bmatrix} \delta \alpha_{i1} \\ \delta \beta_{i1} \\ \vdots \\ \delta \alpha_{in} \\ \delta \beta_{in} \end{bmatrix}, \quad (13)$$

where,

$$\Sigma_i(t) = \Sigma_i(\mathcal{I}_i(t), t) = -C_i(t)$$

$$\begin{bmatrix} M_{i1}(t) \\ \vdots \\ M_{in}(t) \end{bmatrix}^\dagger \begin{bmatrix} M_{i1}(t) C_i(t)^T S(p_1(t) - p_i(t)) \\ \vdots \\ M_{in}(t) C_i(t)^T S(p_n(t) - p_i(t)) \end{bmatrix}$$

$$\Delta_i(t) = \Delta_i(\mathcal{I}_i(t), t) =$$

$$-C_i(t) \begin{bmatrix} M_{i1}(t) \\ \vdots \\ M_{in}(t) \end{bmatrix}^\dagger \text{diag} \left(\begin{bmatrix} M_{i1}(t) C_i(t)^T \\ \vdots \\ M_{in}(t) C_i(t)^T \end{bmatrix} \right)$$

$$\Theta_i(t) = \Theta_i(\mathcal{I}_i(t), t) =$$

$$-C_i(t) \begin{bmatrix} M_{i1}(t) \\ \vdots \\ M_{in}(t) \end{bmatrix}^\dagger \text{diag} \left(\begin{bmatrix} z^{b^T}(p_1(t) - p_i(t)) \\ z^{b^T}(p_1(t) - p_i(t)) \\ \vdots \\ z^{b^T}(p_n(t) - p_i(t)) \\ z^{b^T}(p_n(t) - p_i(t)) \end{bmatrix} \right)$$

Notice that $\Delta_i(t)$ is a block diagonal matrix. We will use the notation $(\Delta_i(t))_k$ for the k block element of its diagonal, i.e.,

$$(\Delta_i(t))_k = -C_i \left(\sum_{j=1}^n M_{ij}^T M_{ij} \right)^{-1} M_{ik}^T M_{ik} C_i^T$$

The supra-index in δp_i^c denotes that this is the error in position calculation obtained with the camera, and should not be confused with δp_i , which is the position error of the INS.

The method for calculating the position of vehicle i with a camera has three different sources of error. The first term (which depends on ϕ_i) is due to the attitude error given by the navigation system of vehicle i . The second term (which depends on δp_j) represents the uncertainty in the position of vehicle j , given by the navigation system of vehicle j . The last source (which depends on $\delta\alpha_{ij}$ and $\delta\beta_{ij}$) is given by the errors in the determination of points α_{ij} and β_{ij} , i.e., a camera error measurement.

V. INTEGRATED NAVIGATION

Navigation systems based on GPS and INS have complementary characteristics. While GPS-based systems allow to bound INS errors, the latter provides navigation information between GPS data acquisitions or when satellites are not available. This motivated the use of both sources of information in the same integrated navigation system. The ability to combine different information sources is not limited to INS and GPS. It is possible to incorporate attitude or position information from radar, altimeters, magnetometers, cameras, etc.

In a nutshell, integrated navigation systems can be described as follows: The INS provides navigation information affected with errors (due to the uncertainty in the initial conditions or to errors in the accelerometers and rate gyros). These errors are expected to be small enough for relative short periods of time. The system errors behave as described in equation (5). Given such model, we can obtain a filter that estimates the INS errors and compensates for them. Therefore, a measurement of such errors is needed. Here is when the additional sensors—such as GPS, magnetometers or cameras—come into play. These additional sensors are usually called *external navigation sensors*, since their measurements depend on external parameters (satellite signals, visual patterns, earth magnetic field, among others).

A. INS/GPS integrated navigation system

Assume that we have $n > 2$ MAV flying and suppose that, additionally to the INS, each vehicle is equipped with a GPS receiver providing its position and velocity. Then, for vehicle $i = 1, \dots, n$ it is possible to implement an integrated navigation system merging the information provided by the INS and the GPS. Here we propose the classical *loosely-coupled* INS/GPS (see for instance [3]). Let $p_i^{gps}, v_i^{gps} \in \mathbb{R}^3$ be the position and velocity given by the GPS, which are corrupted by zero-mean Gaussian white noises η_{p_i} and η_{v_i} with covariance matrices $(\sigma_i^p)^2 I_{3 \times 3}, (\sigma_i^v)^2 I_{3 \times 3} > 0$, respectively. We define the Kalman filter input vector as

$$y_i^{gps} := \begin{bmatrix} \hat{p}_i - p_i^{gps} \\ \hat{v}_i - v_i^{gps} \end{bmatrix} = \begin{bmatrix} \delta p_i + \eta_{p_i} \\ \delta v_i + \eta_{v_i} \end{bmatrix}.$$

Suppose that $\{t_k\}_{k \in \mathbb{N}}$ are discrete times when a measurement of GPS is available. According to equation 5, the state model of the Kalman filter of the integrated navigation system is given by

$$\begin{bmatrix} \delta p_i(t_{k+1}) \\ \delta v_i(t_{k+1}) \\ \phi_i(t_{k+1}) \end{bmatrix} = A_i(t_k) \begin{bmatrix} \delta p_i(t_k) \\ \delta v_i(t_k) \\ \phi_i(t_k) \end{bmatrix} + B_i(t_k) \xi_i(t_k), \quad (14)$$

$$y_i^{gps}(t_k) = H_i^{gps}(t_k) \begin{bmatrix} \delta p_i(t_k) \\ \delta v_i(t_k) \\ \phi_i(t_k) \end{bmatrix} + \eta_i^{gps}(t_k), \quad (15)$$

where $H_i^{gps}(t_k) = \begin{bmatrix} I_{3 \times 3} & 0 & 0 \\ 0 & I_{3 \times 3} & 0 \end{bmatrix}$, for every t_k and $\eta_i^{gps}(t_k) = \begin{bmatrix} \eta_{p_i}(t_k) \\ \eta_{v_i}(t_k) \end{bmatrix}$.

B. INS/Camera integrated navigation system

The scheme proposed for integrating the camera measurement with the INS is similar to that used for the GPS measurements. Although the integration could be done using bearings to each vehicle instead of solving equation (11), in order to use the same formulation as we did for the (loosely-coupled) GPS, we use the vehicle position measured with the camera instead of bearings to each vehicle. But it should be taken into account that a better performance of the system could be achieved if bearings to each vehicle is used for integrating the camera and pseudorange and delta-pseudorange for the GPS.

Suppose that, at time t_k , vehicle i has k_i (different) vehicles $\{p_{j_r}\}_{r=1, \dots, k_i}, j_r \in \mathcal{I}_i$ in its field of view, and satisfies the condition of Remark IV.1. Applying equation (11) we can calculate position p_i . We call this measurement p_i^c , which is corrupted by an error δp_i^c , given by equation (12). We define the input vector as,

$$y_i^c := (\hat{p}_i - p_i^c) = (\delta p_i - \delta p_i^c).$$

Observe that, δp_i^c does not depends only on $X_i = (\delta p_i^T, \delta v_i^T, \phi_i^T)$ but also on every $X_{j_r}, j_r \in \mathcal{I}_i, r = 1, \dots, k_i$. Furthermore, since the vehicles in the field of view of vehicle i change with time, then all states $X_i, i = 1, \dots, n$ must be taken into account in the Kalman filter model. In fact, by equation (13),

$$y_i^c(t_k) = [H_{i1}^c \quad \dots \quad H_{in}^c] \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix} + \Theta_i \begin{bmatrix} \delta\alpha_{i1} \\ \delta\beta_{i1} \\ \vdots \\ \delta\alpha_{in} \\ \delta\beta_{in} \end{bmatrix} \quad (16)$$

where,

$$H_{ij}^c(t_k) = \begin{cases} \begin{pmatrix} I & 0 & \Sigma_i(t_k) \end{pmatrix} & \text{if } i = j \\ \begin{pmatrix} (\Delta_i(t_k))_j & 0 & 0 \end{pmatrix} & \text{if } i \neq j \end{cases} \quad (17)$$

By equation (6), the state model of the Kalman filter for the INS/Camera integrated navigation system is,

$$X(t_{k+1}) = A(t_k)X(t_k) + B(t_k)\xi(t_k) \quad (18)$$

$$y_i^c(t_k) = H_i^c(t_k)X(t_k) + \Theta_i(t_k)\eta_i^c(t_k), \quad (19)$$

where $\eta_i^c(t_k) = [\delta\alpha_{i1}(t_k), \delta\beta_{i1}(t_k), \dots, \delta\alpha_{in}(t_k), \delta\beta_{in}(t_k)]^T$ are assumed zero-mean Gaussian random noises with covariance matrix $(\sigma_i^c)^2 I_{2n \times 2n} > 0$, and

$$H_i^c(t_k) = \begin{bmatrix} H_{i1}^c & H_{i2}^c & \dots & H_{in}^c \end{bmatrix}_{(t_k)}.$$

Fusing both integration strategies, we obtain an INS/GPS/camera integrated navigation system.

VI. BEST FORMATION FLYING CONFIGURATION UNDER DIFFERENT QUALITY NAVIGATION SENSORS

Suppose that we have a formation of n vehicles where $m < n$ of these vehicles have low-cost navigation systems and $n - m$ vehicles are carrying high-quality navigation systems. If the vehicles are performing a certain task, the problem we address in this section is the following: How to distribute the vehicles in the formation in order to achieve the best navigation performance for the whole swarm. More specifically, where should the $n - m$ vehicles with the high quality navigation system be located.

To study this problem, a performance criteria must be established. Observe that, for vehicle $i = 1, \dots, n$, the dynamic model for the Kalman filter of the integrated navigation algorithm is given by (see equation (6))

$$X(t_{k+1}) = A(t_k)X(t_k) + B(t_k)\xi(t_k) \quad (20)$$

$$Y(t_k) = H(t_k)X(t_k) + D(t_k)\eta(t_k), \quad (21)$$

where $Y^T(t_k) = [y_1^T(t_k), y_2^T(t_k), \dots, y_n^T(t_k)]$ are the external measurements of each one of the n vehicles and $\eta^T(t_k) = [\eta_1^T(t_k), \eta_2^T(t_k), \dots, \eta_n^T(t_k)]$ the corresponding (uncorrelated) noises with zero-mean and (diagonal) covariance matrix $R(t_k) = \text{diag}(R_1(t_k), R_2(t_k), \dots, R_n(t_k)) > 0$. More precisely, for $i = 1, \dots, n$:

$$y_i = \begin{bmatrix} y_i^{gps} \\ y_i^c \end{bmatrix} \in \mathbb{R}^9, \eta_i = \begin{bmatrix} \eta_i^{gps} \\ \eta_i^c \end{bmatrix} \in \mathbb{R}^{2n+6},$$

$$R_i(t_k) = \begin{bmatrix} (\sigma_i^p)^2 I_{3 \times 3} & 0 & 0 \\ 0 & (\sigma_i^v)^2 I_{3 \times 3} & 0 \\ 0 & 0 & (\sigma_i^c)^2 I_{2n \times 2n} \end{bmatrix} > 0,$$

where $(\sigma_i^p)^2, (\sigma_i^v)^2 > 0$ stands for GPS position and velocity variance noises and $(\sigma_i^c)^2 > 0$ for the camera variance noise, respectively. Notice that here it is assumed that the variance noise is the same in x, y and z -axis.

Matrix $D(t_k)$ is a block diagonal, $D(t_k) = \text{diag}(D_1(t_k), D_2(t_k), \dots, D_n(t_k))$, where

$$D_i(t_k) = \begin{bmatrix} I_{6 \times 6} & 0 \\ 0 & \Theta_i(t_k) \end{bmatrix} \in \mathbb{R}^{9 \times (2n+6)},$$

for $i = 1, \dots, n$. In what follows we assume that $\Theta_i(t_k)$, and hence $D_i(t_k)$, are full rank matrices for all $i = 1, \dots, n$ and $t_k \geq t_0$.

Matrix $H(t_k) = [H_1^T(t_k), H_2^T(t_k), \dots, H_n^T(t_k)]^T$, where

$$H_i(t_k) = \begin{bmatrix} 0 & \dots & H_i^{gps}(t_k) & \dots & 0 \\ & & H_i^c(t_k) & & \end{bmatrix} \in \mathbb{R}^{6 \times 9n},$$

for $i = 1, \dots, n$.

A. Kalman filter covariance and information matrices

Let $P(t_k)$ and $P^-(t_k)$ be the update and prediction covariance matrices of the Kalman filter corresponding to the formation (see equations (20)). More precisely, if $\hat{X}(t_k)$ is the Kalman filter estimate of state $X(t_k)$ given measurements $\{t_i\}_{i=0, \dots, k}$, then

$$P(t_k) = E((X(t_k) - \hat{X}(t_k))(X(t_k) - \hat{X}(t_k))^T).$$

In a similar way, if $\hat{X}^-(t_k)$ is the Kalman filter prediction of state $X(t_k)$ given measurements $\{t_i\}_{i=0, \dots, k-1}$, then

$$P^-(t_k) = E((X(t_k) - \hat{X}^-(t_k))(X(t_k) - \hat{X}^-(t_k))^T).$$

Matrices $P(t_k), P^-(t_k)$ are assumed to be strictly positive. Also denote $I(t_k) = P(t_k)^{-1}$ and $I^-(t_k) = P^-(t_k)^{-1}$, which are the update and prediction information matrices. It is well-known that the relation between $I(t_k)$ and $I^-(t_k)$ is given by [15],

$$I(t_k) = I^-(t_k) + H(t_k)^T (D(t_k)R(t_k)D(t_k)^T)^{-1} H(t_k). \quad (22)$$

These matrices give a measure of the navigation performance of the formation. In fact, the trace of matrix $I(t_k)$ is closely related to the observability of state $X(t_k)$ [16]. If matrix $I(t_k)$ increases with time (in the order given for positive matrices) then the navigation error decreases. Besides the trace of the information matrix, there are several scalar measures of how well the system performs; see for instance [17], which is a survey of different scalar performance measures for tracking and sensor allocation problems.

One of the main reasons to work with the information matrix in navigation is that it allows to separate information coming from each sensor. This fact is used in the next section to select the optimal vehicle formation for the MAV. However, other measures based on the covariance matrix have a more clear physical interpretation. The A-optimality and D-optimality criteria decide the best filter solution based on the trace and determinant of the covariance matrix, respectively [17].

It is possible to estimate a solution for the D-optimality criteria based on the trace of information matrix [18].

B. MAV Formation flying

In this section we assume that n vehicles are performing a given task that requires such vehicles to preserve their attitude and relative positions, i.e., $p_i(t_k) - p_j(t_k) = p_i(t_l) - p_j(t_l)$ for all $i \neq j = 1, \dots, n$ and times $t_k, t_l > t_0$.

As mentioned before, suppose that some vehicles have a better navigation system, more specifically, better external navigation sensors but similar inertial sensors. We will assume that the vehicles are numbered with respect to their external sensors quality, i.e., $\sigma_1^p \leq \sigma_2^p \leq \dots \leq \sigma_n^p$, also $\sigma_1^v \leq \sigma_2^v \leq \dots \leq \sigma_n^v$, and $\sigma_1^c \leq \sigma_2^c \leq \dots \leq \sigma_n^c$.

Notice that, if inertial sensors are similar and their errors are assumed to be stationary, it follows that $Q = Q_i(t_k) = E(\xi_i(t_k)\xi_i(t_k)^T) > 0$. Thus, $E(\xi(t_k)\xi(t_k)^T) = \text{diag}(Q, \dots, Q)$.

The problem we are interested in is: Does the navigation quality of the formation depend on where each vehicle is located in the formation? As a quality index of the formation navigation performance we use the trace of information matrix $I(t_k)$.

Note that relocation of vehicles in the formation can be seen as a reordering the diagonal blocks $R_i(t_k)$ of matrix $R(t_k) = \text{diag}(R_1(t_k), R_2(t_k), \dots, R_n(t_k))$. Each formation has associated an information matrix. In fact, each reordering of $R(t_k)$ can be represented as $\Pi^T R(t_k) \Pi$, where Π is a block permutation matrix. Thus we can give a parametrization of the set of information matrices corresponding to the different formations. Notice that, at most we have $n!$ different information matrices:

$$I_\Pi(t_k) = I_\Pi^-(t_k) + H(t_k)^T (D(t_k) \Pi^T R(t_k) \Pi D(t_k)^T)^{-1} H(t_k). \quad (23)$$

Since $D(t_k) \Pi^T R(t_k) \Pi D(t_k)^T$ is a block diagonal matrix, it follows that,

$$I_\Pi(t_k) = I_\Pi^-(t_k) + \sum_{j=1}^n H_j(t_k)^T (D_j(t_k) \Pi^T R_j(t_k) \Pi D_j(t_k)^T)^{-1} H_j(t_k).$$

Let $\mathcal{I} = \{1, \dots, n\}$ and $f_\Pi : \mathcal{I} \rightarrow \mathcal{I}$ a bijective function, each of these functions corresponding to a different block permutation matrix Π . Since $H_i^{gps} = H_j^{gps}$, for every $i, j = 1, \dots, n$, by equations (14) and (18), it follows that,

$$I_\Pi(t_k) = I_\Pi^-(t_k) + \text{diag}(J_1, \dots, J_n) + \sum_{j=1}^n (\sigma_{f_\Pi(j)}^c)^{-2} H_j^c(t_k)^T (\Theta_j(t_k) \Theta_j(t_k)^T)^{-1} H_j^c(t_k), \quad (24)$$

where, for $j = 1, \dots, n$

$$J_j = \begin{bmatrix} (\sigma_{f_\Pi(j)}^p)^{-2} I_{3 \times 3} & 0 & 0 \\ 0 & (\sigma_{f_\Pi(j)}^v)^{-2} I_{3 \times 3} & 0 \\ 0 & 0 & 0_{3 \times 3} \end{bmatrix}$$

The second term of equation (24) shows that the location of the vehicles with better GPS receivers does not affect the formation navigation performance, because the trace of this term is invariant respect any vehicle permutation. However, a better camera sensor can affect the navigation performance. This can be studied looking at the third term of the equation above. This is an expected result since the camera gives relative vehicle positions, meanwhile GPS gives absolute position information.

Now, if attitude and relative vehicle positions are fixed, then $H_j = H_j(t_k)$ and $\Theta_j = \Theta_j(t_k)$ do not depend on t_k . From equation (24), it follows that

$$\text{tr}(I_\Pi(t_k)) = \text{tr}(I_\Pi^-(t_k)) + 3 \sum_{i=1}^n \left((\sigma_i^p)^{-2} + (\sigma_i^v)^{-2} \right) \quad (25)$$

$$+ \sum_{j=1}^n \left(\sigma_{f_\Pi(j)}^c \right)^{-2} \text{tr} \left(H_j^c(t_k)^T (\Theta_j \Theta_j^T)^{-1} H_j^c(t_k) \right) \quad (26)$$

We are interested in solving the following problem: Given $I_\Pi^-(t_k)$ find the argument of

$$\max_{\{f_\Pi : \mathcal{I} \rightarrow \mathcal{I}, \text{ bijective} \}} \text{tr}(I_\Pi(t_k))$$

Solutions to this problem indicate where the vehicles with better camera sensors should be located in order to achieve the best navigation performance for the formation. Although the solution above can be found solving $n!$ Kalman filters, this is numerical intractable if we have several vehicles in the formation. The following results allows us to find the solution to the problem presented above.

Lemma VI.1. *Let $A_1, A_2, \dots, A_n \in \mathbb{R}^{n \times n}$ be positive matrices such that $\text{tr}(A_1) \geq \text{tr}(A_2) \geq \dots \geq \text{tr}(A_n)$, and let $\alpha_1 \geq \alpha_2 \geq \dots \geq \alpha_n \geq 0$. Then,*

$$\text{tr} \left(\sum_{i=1}^n \alpha_i A_i \right) \geq \text{tr} \left(\sum_{i=1}^n \beta_i A_i \right),$$

for every $(\beta_1, \beta_2, \dots, \beta_n) = f_\Pi((\alpha_1, \alpha_2, \dots, \alpha_n))$.

Proof: The proof follows from the linearity of trace operator and from the fact that $\text{tr}(A_i) \geq 0$, since $A_i \geq 0$.

It makes sense to study the *update-step* in the information filter, because different quality sensors are fundamental in this step. Inertial sensors are involved in the *prediction-step*, and every vehicle is carrying a similar inertial navigation system.

The results presented above allow us to give a positive answer to the question that motivates this section.

Theorem VI.2. *Suppose we have a formation of $n \in \mathbb{N}$ vehicles such that, for every $t_k, t_l > t_0$, $p_i(t_k) - p_j(t_k) = p_i(t_l) - p_j(t_l)$ and $C_i(t_k) = C_i(t_0)$, with $i, j = 1, \dots, n$. Suppose that the external sensors satisfy $0 < \sigma_1^c \leq \sigma_2^c \leq \dots \leq \sigma_n^c$, and inertial sensors errors satisfy $Q = Q_i(t_k) = E(\xi_i(t_k) \xi_i(t_k)^T) > 0$, for every $i = 1, \dots, n$. Then, the best formation flying configuration satisfies*

$$\text{tr} \left(H_n^c(t_k)^T (\Theta_n \Theta_n^T)^{-1} H_n^c(t_k) \right) \leq \dots \leq \text{tr} \left(H_1^c(t_k)^T (\Theta_1 \Theta_1^T)^{-1} H_1^c(t_k) \right)$$

Proof: The proof follows from Lemma VI.1 and equation (25).

VII. SIMULATION RESULTS

In this section we present simulation results to validate the proposed INS/GPS/camera integration strategy. First, we show how relative position information improves the navigation performance, for this a benchmark trajectory is proposed to analyse the incidence of the camera measurements on the navigation performance.

The second simulation shows how the navigation performance of the formation is modified when we rearrange the order of the vehicles. We show a simple (but realistic) example to put in evidence the importance of the location of each vehicle.

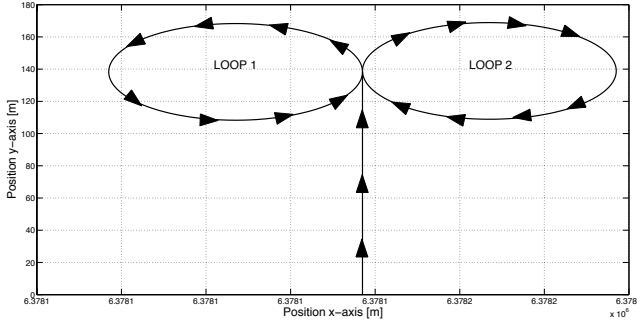


Fig. 2. Trajectory of one MAV with respect to an initial position: straight flight (0 s to 200 s), left turn (200 s to 335 s), right turn (335 s to 470 s).

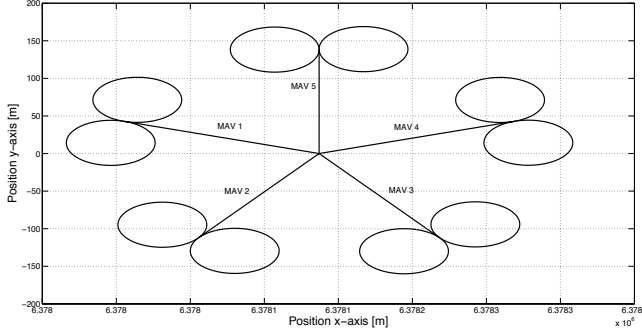


Fig. 3. Trajectories of the 5 MAVs

A. Navigation system comparison

The first case of study is a formation of 5 MAVs flying with constant altitude (1.2 m) above the ground and following the trajectory shown in Figure 2. Each vehicle follows the same trajectory relative to the initial point but with a different initial attitude, as shown in Figure 3.

The vehicles have INS/GPS/camera navigation systems with the following specifications:

- INS:
 - Rate gyros noise: 1.5×10^{-3} rad/s (σ).
 - Accelerometers noise: 5×10^{-3} m/s² (σ).
 - Initial attitude error: 10° (randomly distributed).
- GPS:
 - Position noise: 15 m (σ).
 - Velocity noise: 0.2 m/s (σ).
- Camera:
 - Measurement noise: 0.5° (σ) (equivalent).
 - Mask: 70° (view angle from MAV nose).

In the following section, we present the navigation results only for vehicle 1 (MAV1) since similar results hold for the remaining vehicles.

Figures 4 and 5 show position and attitude error corresponding to the navigation system of MAV1 for 25 realizations. Figure 6 shows the vehicles that are in the field of view of MAV1 over time.

It can be seen that around $t = 225$ s the first vehicle is detected (vehicle 5). In Figure 5, it can be noted at that time, how the attitude error calculated by the navigation system is

reduced since relative information is included. Furthermore, it can also be seen that around $t = 320$ s the quality of the estimation starts degrading, in agreement with the period in Figure 6 when no vehicles are in the camera's field of view. This illustrates the improvement to the navigation system due to the use of the camera.

B. Vehicles optimal allocation

In this section we study, for a given trajectory, how the navigation performance of the formation is affected by different vehicles arrangements. More precisely: there are 5 MAV flying in formation and one of them is assumed to have a better navigation system. The vehicles are in hovering position maintaining a fixed formation (equally distributed in a circle), as it is shown in Figure 7. Only vehicle MAV1 has camera information, the remaining vehicles only have INS/GPS navigation information. We locate the vehicle with better navigation system (MAV1) in different positions (more specifically we switch MAV1 and MAV2) and we study how the navigation performance is affected. It is not difficult to predict the outcome of this simulation since when the camera is installed in MAV1, four vehicles are in the field of view. However, when camera sensor is placed in MAV2 no vehicles are in the field of view (because of the attitude of MAV2). In this example it is clear that an optimal solution for vehicle location is to place the vehicle with the camera in position MAV1 (according to Figure 7). This is the solution given in Theorem VI.2. When the camera is located in a vehicle different from MAV1 then, $\text{tr}(H_j^c T (\Theta_j \Theta_j^T)^{-1} H_j^c) = 0$, for every $j = 1, \dots$ since there are no vehicles in the field of view of the camera. However, if the camera is located in MAV1 then, $\text{tr}(H_j^c T (\Theta_j \Theta_j^T)^{-1} H_j^c) \neq 0$, for every $j = 2, \dots$. Thus, if $I_0 \geq 0$ is the information matrix corresponding to the ordering with the camera located in the position of MAV1, then according to equation (25), it follows that $\text{tr}(I_0) \geq \text{tr}(I_\Pi)$, for every vehicle reordering Π .

Figures 8 and 9 show the effect on attitude error of the navigation system when we switch MAV1 and MAV2. Figure 8 shows the attitude error of MAV1 when the camera is located in MAV1, the attitude error of the remaining vehicles has a behavior similar to Figure 9. On the other hand, if camera is located in MAV2 then the attitude error of all vehicles is similar to Figure 9.

This is a simple example because only one vehicle is equipped with a camera and it is easy to see where it should be located. But the idea can be extended to a more complex vehicle formation.

It is worth to mention that the idea presented here can be used for different navigation configurations, however here it is presented for a loosely coupled INS+GPS+Camera configuration. For some sensors, in particular for camera sensor, in real application the assumption that some cameras are better than others (i.e., $\sigma_1^c \leq \dots \leq \sigma_n^c$) is not well-founded. But even in the case, Theorem two-optm-alloc states where should be located the vehicles carrying the cameras in order to make the most of this information, as previous example shows.

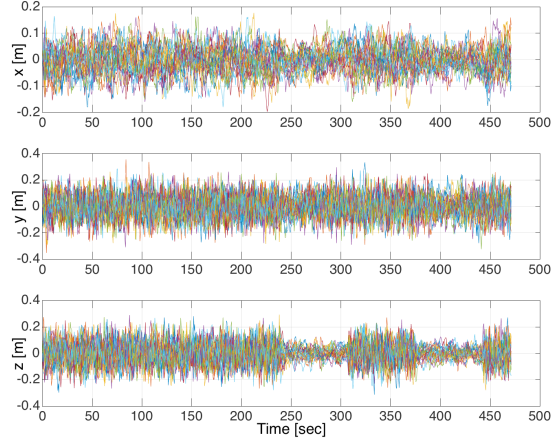


Fig. 4. Position error of the navigation system of MAV1. Similar results hold for the others vehicles.

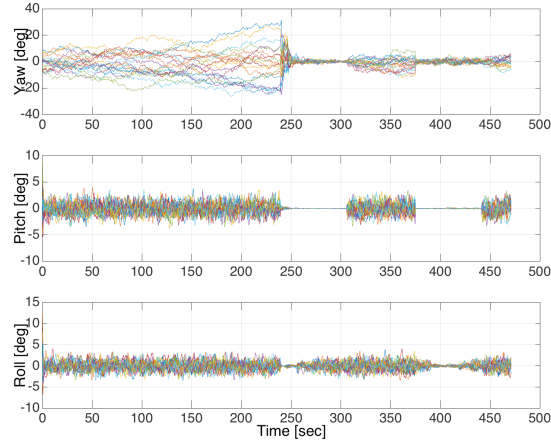


Fig. 5. Attitude error of the navigation system of MAV1. Similar results hold for the others vehicles.

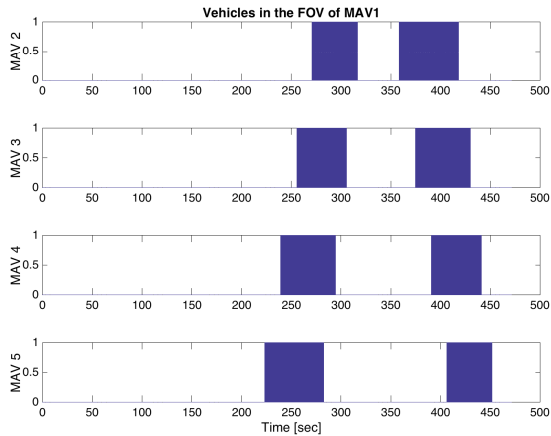


Fig. 6. Vehicles in the field of view of camera 1 over time

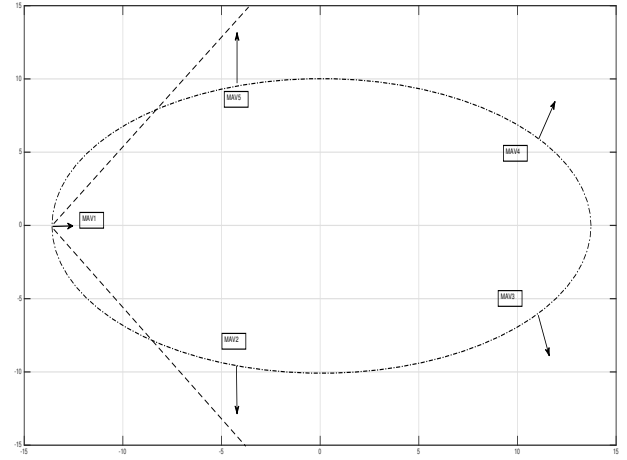


Fig. 7. Vehicles in formation. Arrows show where is pointing the camera of each vehicle. Only MAV1 has in the field of view the others vehicles in the formation, as dotted lines show.

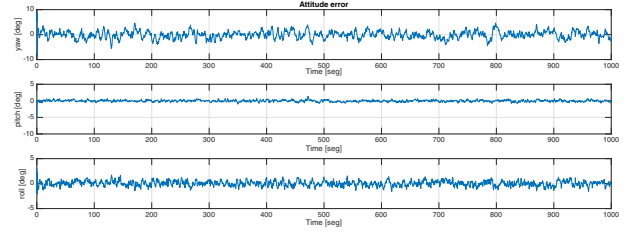


Fig. 8. Attitude error for the optimal ordering.

VIII. CONCLUSIONS

In this article we presented an integrated navigation algorithm for a fleet of UAV. Each vehicle carries a navigation system that fuses information of inertial sensors, GPS and a camera which provides position information of the surrounding vehicles. It is assumed that each vehicle is capable of transmitting its current navigation position. Also it is assumed that each vehicle can be identified with the camera.

The use of relative positioning between vehicles has been widely studied in the literature, not only for navigation but for implementing control algorithms for formation flying. From the point of view of navigation, several interesting problems arise. In particular we are interested in studying how the navigation performance is affected if we reorder the vehicles in the formation. More specifically, we assumed that some vehicles in the formation have better quality navigation systems and we study where these vehicles should

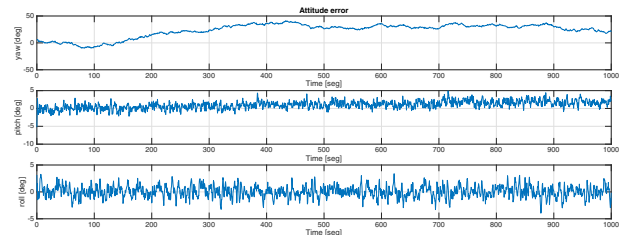


Fig. 9. Attitude error for the non-optimal ordering.

be located to maximize the benefit of their high-quality navigation information. Although this problem can be solved numerically, we were interested in obtaining a formal result that gives the optimal formation. In order to obtain this result, we define a metric for the navigation performance of the fleet based on the trace of the associated information matrix. Under certain conditions, we showed that the optimal formation can be found comparing the model measurement matrices corresponding to each exterioreceptive sensor (GPS or camera). It is worth mentioning that although we focus on GPS or camera sensors, this result can be extended to others exterioreceptive sensors that provide relative vehicle position information.

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