Application of an Optimized LEB Filter to INS/GPS Test Data

Claudio D Antonini

Buenos Aires Institute of Technology
Electronics Engineering Department
Buenos Aires, Argentina

University of Pretoria
Electrical and Electronics Engineering
Pretoria, South Africa

Abstract

An optimized Linear-Ellipsoidal-Bounded (LEB) filter has been developed and applied to data obtained from a ground test using a combined INS/GPS configuration. In this cascaded configuration, the filter receives eight outputs from the INS (accelerations, velocity, angles, altitude) and six outputs from the GPS (velocities and positions). The GPS measurements have included the effect of SA –of varying or unknown spectrum – which, although likely to be estimated and compensated with some modelling techniques –at the expense of including extra state variables–, could also be dealt with the approach indicated in this article at much less effort. The SA effect is modelled as an unknown-but-bounded (UBB) noise process. Comparisons with an Extended Kalman filter (KF) show that KF innovations are not white and the LEB filter innovations are one order of magnitude smaller that those produced by the KF. A simple second order example is developed to show the behavior of the LEB filter when compared to a KF.

Introduction

Prior work on the INS/GPS systems can be found in [2] or [1]. Basically, the structures considered to find estimates of velocities and position consist of an INS pack, a GPS and a filter that combines the outputs of the previous two subsystems. The structures differ in the parameters selected as outputs of the GPS. The outputs available to the filter could be raw outputs (orbital parameters and pseudoranges) or processed outputs (velocities and positions). As a first step in using this system, we decided not to use the raw data but to use the velocities and positions as already processed by the GPS in a cascaded configuration, approach that would make it useful for compatibility with almost any other GPS equipment.

The LEB filter was originally introduced in [9] and cited in [4]. However, not much was published in the open literature extending (or even quoting) the original formulation of [9] or [10]. An extension of the LEB filter that minimizes the volume of the ellipsoid containing the estimation errors was developed in [3].

Very few results can also be found related to the modelling of SA. Worth mentioning are [5] – who proposed two LTI models for the SA effect – and [8] –where a simplified model of [5] was incorporated in an Extended KF–.

These solutions attempt to model the effect of SA by increasing the number of states in the dynamic model of the INS/GPS. The reason behind this approach is that, for a given autocorrelation function, there always exist a gaussian random process with the same autocorrelation function [7]. Thus, if the noise input is modelled as a zero mean white stochastic process then the modification of the spectrum is left to a shaping filter, being it necessary to augment the dynamic model of the plant.

Our approach, on the contrary, keeps constant the number of states by considering that the modification may be incorporated in our assumptions about the noises, in this case, the measurement noise.

The INS/GPS model

The sensors used in the test correspond to an INS pack and a GPS equipment. The INS section of the sensors uses three accelerometers from which we obtain the three accelerations in body axis ($a_{x_m}, a_{y_m}, a_{z_m}$), an ADS (Air Data System) providing altitude above sea level (alt_m and airspeed $vias_m$), a vertical gyro whose outputs are roll angle (Φ_m) and pitch angle (Θ_m), and a magnetometer which outputs magnetic heading (Ψ_m). The outputs of the GPS are three estimated velocities in LLLN (Local Level Local North) axis (v_n, v_e, v_d) and three estimated positions (L, l, alt). This model has been fully described in [3].

Although the structure of the dynamic model is the same in both the LEB filter and the KF approach, the interpretations are different: in the LEB approach the inputs (noises) and the states do not need any probabilistic interpretation. They are processes whose only characteristic is that they are bounded: the numerical values for these physical variables will have a minimum and a maximum and, at any time t, the variables will be found between those lower and upper bounds, respectively. Thus, for example, from [3], the first order Markov process that defines the bias in the altimeter, ($=alt_b$), valid for a KF, is simply a first order bounded process in the LEB interpretation [6].

In order to assign standard deviations to the noises in the KF, it was assumed that three standard deviations correspond to the maximum values of the noises. Thus, for example, the spectral density of the (now, white) noise in the GPS North velocity measurement will be $v_{v_n}^2/3^2 = 1\,[m^2/s^3]$, if the maximum (minimum) bound for the LEB filter is $\pm 3[m/s]$.

The LEB Filter and the optima β and α

Given the usual representation for discrete-time linear systems

$$x_{(+)} = \Phi_{(-)}x_{(-)} + G_{(-)}w_{(-)} \quad (1)$$
$$y_{(+)} = C_{(+)}x_{(+)} + v_{(+)} \quad (2)$$

the Linear-Ellipsoidal-Bounded (LEB) formulation can be used to find a filter that generates estimates $\hat{x}_{(+/+)}$ such that they are contained in the ellipsoid defined by $\Sigma_{(+/+)}$ [10]:

$$\hat{x}_{(+/+)} = \Phi_{(+)}\hat{x}_{(-/-)} + K_{(+)}\left\{y_{(+)} - C\Phi_{(+)}\hat{x}_{(-/-)}\right\} \quad (3)$$
$$K_{(+)} = \rho_+\Sigma_{(+/+)}C'R^{-1} \quad (4)$$
$$\Sigma_{(+/+)} = \left\{(1-\rho_+)\Sigma^{-1}_{(+/-)} + \rho_+ C'R^{-1}C\right\}^{-1} \quad (5)$$
$$\Sigma_{(+/-)} = \frac{\Phi\Sigma_{(-/-)}\Phi'}{1-\beta_+} + \frac{GQG'}{\beta_+} \quad (6)$$
$$\Omega_{x_{(+/+)}} = \left\{x : [x - \hat{x}_{(+/+)}]'\Sigma^{-1}_{(+/+)}[x - \hat{x}_{(+/+)}] \leq 1\right\} \quad (7)$$
$$0 < \beta_+ < 1 \quad (8)$$
$$0 < \rho_+ < 1 \quad (9)$$

The optimal β_+ is [3]

$$\beta_+ = \frac{1}{1 + \sqrt{\frac{trace(\Sigma^{-1}_{(+/-)}\Phi\Sigma_{(-/-)}\Phi')}{trace(\Sigma^{-1}_{(+/-)}GQG')}}} \quad (10)$$

and the optimal ρ is found with Newton iterations as follows:

$$\rho^{i+1}_+ = \rho^i_+ - \frac{trace(X)}{trace(X)^2 + trace(X^2)} \quad (11)$$

where

$$i = 1, 2, \ldots, \text{maxit}$$
$$\text{maxit} = \text{maximum number of iterations}$$
$$\rho^1_+ = \rho^*_- \text{ previous optimal } \rho$$
$$X = \left(\Sigma^{-1}_{(+/-)} - C'R^{-1}C\right)\Sigma_{(+/+)} \quad (12)$$

It has to be noted that the estimate $\hat{x}_{(+/+)}$ is a set, defined by $\Omega_{x_{(+/+)}}$. No point of that set is more likely to be the estimate more than any other point of the set. However, it is natural to consider the center of the set as the vector estimate.

If there are no abrupt changes in the noises or in the dynamics of the system, it is not necessary to calculate β_+ and ρ_+ at every cycle of the filter.

The filter has to be allowed to run for a few cycles (in our case, ten cycles was enough) to develop a covariance matrix that would allow $\partial det(\Sigma_{(+/+)})/\partial\rho$ to have a minimum for some ρ between 0 and 1. Otherwise, when started with a diagonal matrix, the minimum was always $\rho = 0$ during the first iterations, [3].

Results of the Ground Test

The test was performed driving on roads for a total length of about 44 km, in August of 1992. The circuit, along with the estimates provided by the LEB and KF, is shown in Figures 1 and 2, respectively, where (0,0) corresponds to the start of the test. Data was captured, processed on line with a KF and offline with the LEB filter.

The results of the test can be seen in Figure 3, where the distances from the estimates to the GPS position fixes is shown. The average difference between the LEB Filter estimate and the GPS position, the innovations in position, is about 10 m while the KF estimates are up to one order of magnitude larger.

Second order example

In order to understand the difference in the behavior between the KF and LEB filters, a simulation of a simple, second order system, was performed. Zero mean, white gaussian process noise was added and, also, two types of measurement noise were considered: the usual zero mean white gaussian spectrum and a square signal of ± 1, switching at every sample. The KF was designed as a fixed-gain filter. The LEB was optimized at every cycle finding the β and ρ to minimize the determinant of the $\Sigma_{(+/+)}$ matrix, according to the procedure given above.

The system selected is the following:

$$A = \begin{bmatrix} -0.1 & 0 \\ 1 & -0.2 \end{bmatrix}; \; B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}; \; C = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

The spectral densities for the process and measurement noises are $q = 0.01$ and $r = 1$, respectively. The initial conditions for the states are zero and for the covariances, the identity matrix. The starting β and ρ are both 0.5. With these initial values, the optimal starting ρ was found iterating the equations (5), (6) and (11) ten times, until succesive ρ's differences were less than 0.0001.

In Figure 4, the states, the estimates and the innovations are shown for the KF and the LEB implementations, the first row for a KF with white noise, the second for a KF with square noise, the third for a LEB filter with white noise and the last one for a LEB filter with the square noise. Both filters were run with constant gains. It can be observed that the LEB estimates -on the average- are closer to the real states than the KF during the whole simulation, for both noises. At the beginning of the simulation, given that the LEB filter, in general, produces larger gains than the KF, the LEB estimates are closer to the real states when there are sudden changes in the system (as shown tracking the initial condition error). Later in the simulation, the KF gives smoother estimates. The behavior of the KF could be changed by modifying the intensity of the noises (in order to track the initial conditions), but the LEB filter gave acceptable results from the start for both measurement noise spectrums, without the need to experiment with the matrices representing the intensity of the noises.

Conclusions

Given that many of the problems in the determination of an optimal structure and solution for configurations that use GPS is the desire of incorporating the effect of SA (Selected Availability), it was decided to include the SA effect as an unknown-but-bounded process with unknown spectrum.

A Kalman filter is probably not the best structure to do so because it is necessary to augment the state vector. If we do not want to increase the dimensions of the matrices of the model because we may be bound by, say, processing time or memory, then we must resort to other assumptions in the derivation of a filter. The assumption of treating SA as an UBB noise achieves this purpose.

In order to minimize the amount of processing, not only the number of states was kept constant by using a LEB filter instead of a KF: the outputs of the INS were included directly in the dynamic model of the plant and appropriate error states were introduced in the model with noises modelled also as UBB processes.

It was found that the innovations of the LEB Filter were one order of magnitude smaller than the KF innovations and that the spectrum of the KF innovations was not white, which would suggest errors in the modelling of the dynamics (according to the usual interpretation, if a KF model is used). In our case, we know that it is not necessary to modify the model if modified assumptions are used for the noises. As we saw, the use of UBB noises definitely improves the results without changing the model.

A second order model was developed and a KF and LEB filter were applied to the system with added measurement noise of different spectrums. This situation would be similar to the addition of SA to our GPS measurements. The LEB filter gave closer estimates to the real -simulated- state variables, on the average, than the KF.

References

[1] NATO, *AGARD-AG-314, Analysis, Design and Synthesis Methods for Guidance and Control Systems*, June 1990.

[2] IEEE, *Position Location and Navigation Symposium*, November 1986, Las Vegas, NV.

[3] C. Antonini, Development of an optimized LEB filter and its application to INS/GPS test data, In *AIAA GNC Conference*, August 1993.

[4] C. Antonini, Performance and Stability Robustness of LEB Compensators, In *XXVII Annual Allerton Conference*, pages 441–449, September 1989.

[5] M. Braasch, A Signal Model for GPS, *Navigation*, 363–377, Winter 1990-1991.

[6] A. Bryson and Y. Ho, *Applied Optimal Control*, Halstead Press, 1975.

[7] A. Gelb (Ed.), *Applied Optimal Estimation*, MIT Press, 1974.

[8] W. Negast and R. Paschall, Compensation of Selected Availability using a GPS/INS Extended Kalman Filter, In *NAECON 92*, pages 356–362, 1992.

[9] F. Schweppe, Recursive State Estimation: Unknown but Bounded Errors and System Inputs, *IEEE-TAC*, 22–29, February 1968.

[10] F. Schweppe, *Uncertain Dynamic Systems*, Prentice-Hall, 1973.

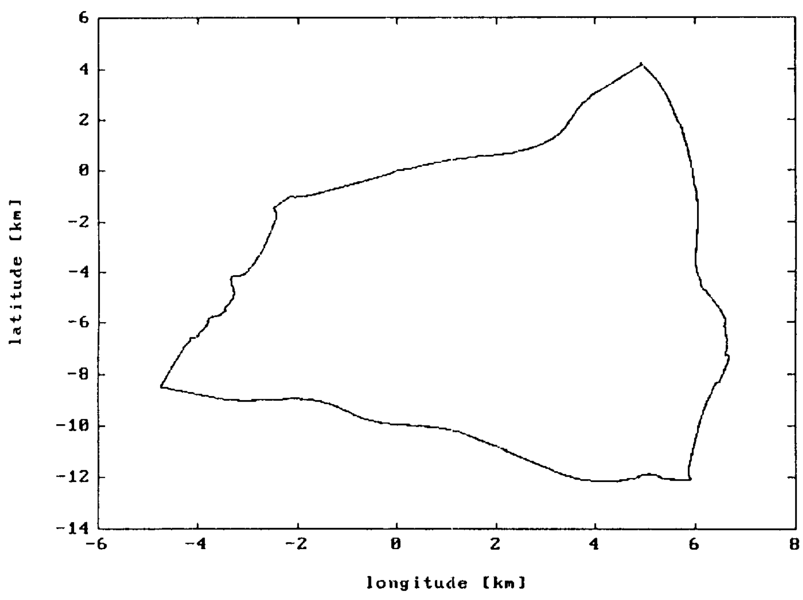

Fig. 1. GPS and LEB Filter estimates.

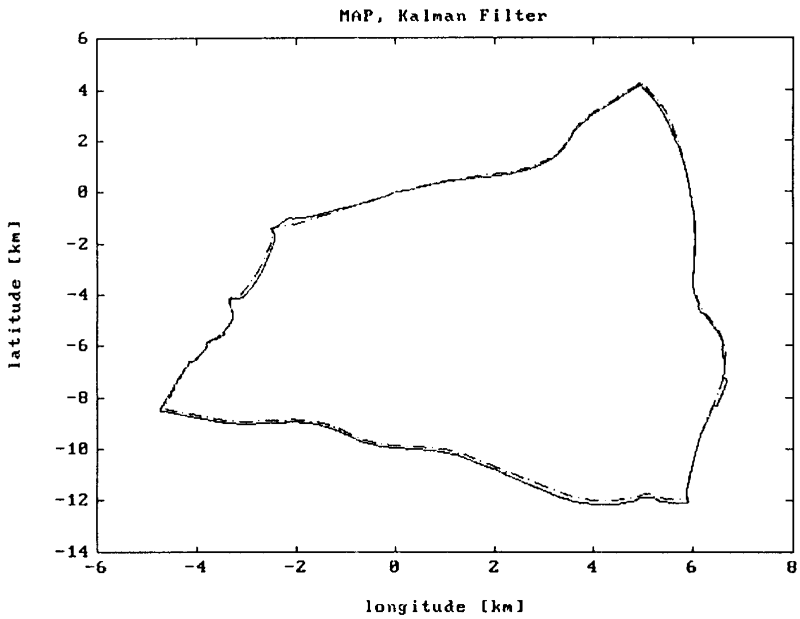

Fig. 2. GPS and Kalman Filter estimates.

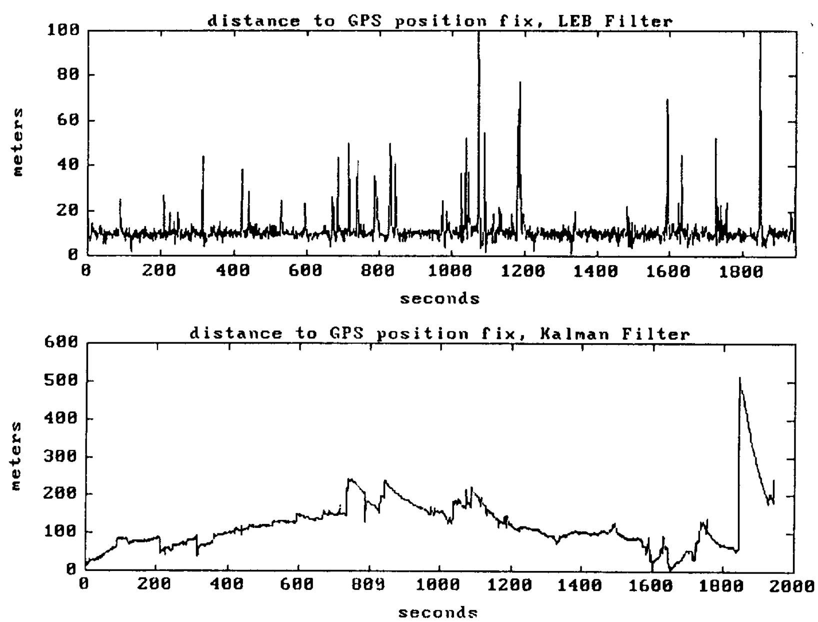

Fig. 3. 3D distances to GPS estimates.

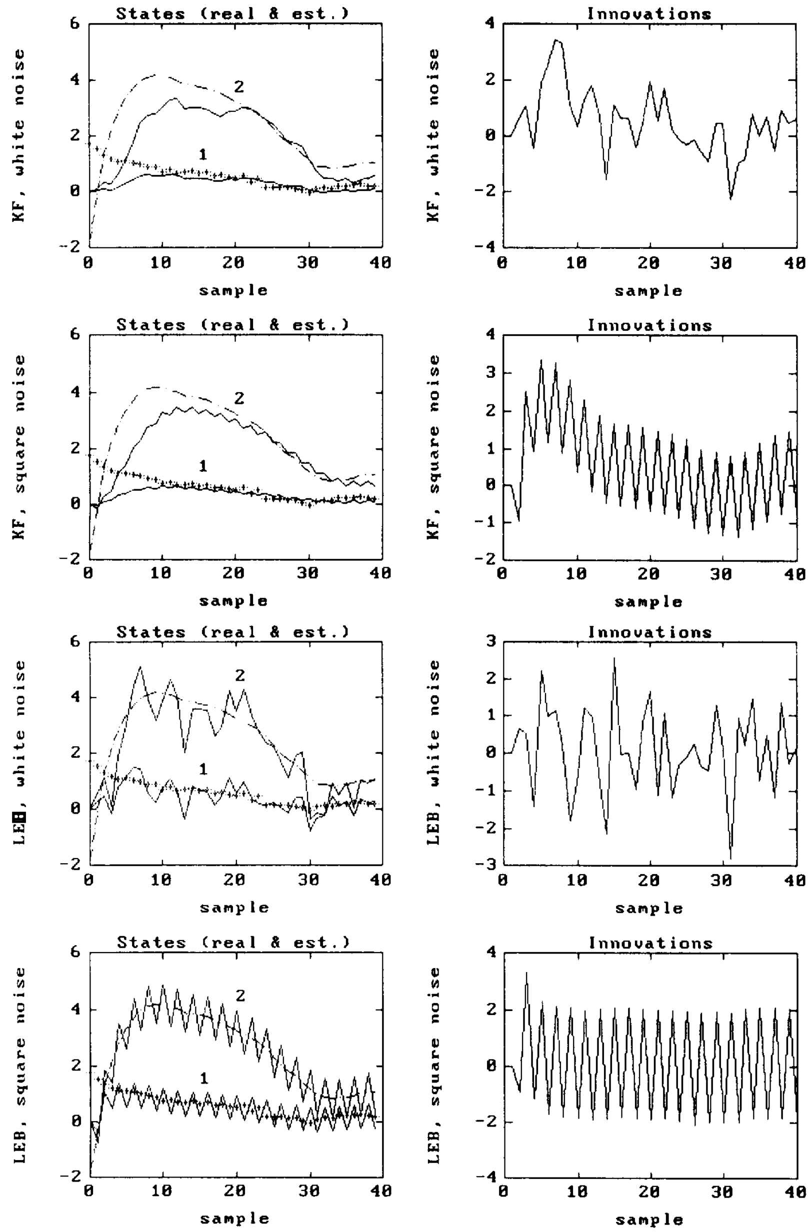

Fig. 4. KF (top two rows) and LEB Filter (bottom two rows) applied to second order system with white noise (first and third rows) and a square signal (second and fourth rows) added as measurement noises.