Elongated self-propelled particles roaming a closed arena present financial stylized facts

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Abstract. We report the existence of financial stylized facts in a system of mechanical vehicles driven by vibration (VDV). The VDVs are restricted to a closed geometry that is composed of two chambers connected by an opening which allows a continuous flow of agents between the two regions. We studied the temporal evolution of the density of particles around the opening and made a statistical comparison with the price evolution of bitcoin (BTC). We found remarkable similarities between these two systems enabling us to study financial systems from a new perspective.

Keywords: self-propelled particles, stylized facts, bitcoin, active matter

1 Introduction

Financial stylized facts are the common statistical properties that can be found in different studies of markets and instruments. Although they are qualitative properties, stylized facts are difficult to find in synthetic stochastic processes generated by artificial models [1]. Recently, it has be shown that the counterflow through a door of simulated pedestrians, with decision-making capacity, presents several of the stylized facts observed in financial markets [2]. This picture of counterflows of particles through a narrow constriction can be interpreted as a flow of two types of orders (buy and sell) that interact in a narrow price interval. In this direction, several physical analogies were proposed to describe different financial properties [3–6].

In this work, we present results of experiments carried out with vehicles driven by vibration [7–9]. The system under study comprises two closed regions linked by a narrow opening that yields a balanced flow with a bottleneck. We studied the temporal evolution of the density of particles around the opening and made a statistical comparison with the price evolution of the Bitcoin cryptocurrency price. Our findings show that the density fluctuations experimentally reproduce the main stylized facts, which allow us to address the study of financial systems from a new perspective.

2 Experiment

We built a closed arena comprised of two regions connected by a narrow opening of length $L = 40$ mm using acetate tape as flexible walls and wooden blocks to fix the opening size. This particular design allows us to generate a continuous flow of particles through a bottleneck. Figure 1(a) shows the experimental setup. We used 13 vibration-driven vehicles (VDVs) named Hexbug Nano [10]. The dimensions of these vehicles are 43 mm × 15 mm × 18 mm. As can be seen in Fig. 1(a), we used a four-color label design for the tracking of the VDVs.

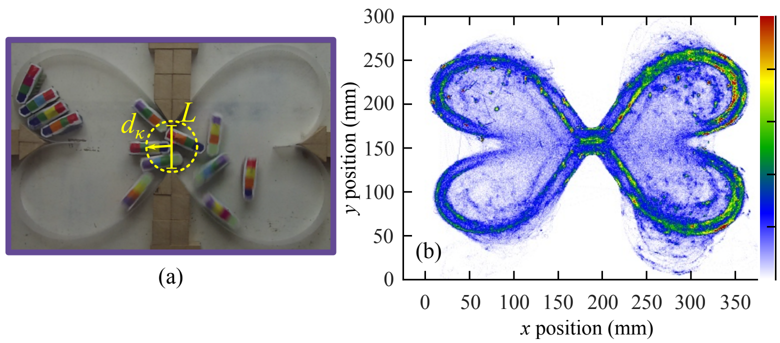

Fig. 1. (a) Experimental setup. Two regions are connected by a narrow opening of size $L = 40$ mm. (b) Estimated PDF of the positions of all agents. A linear color scaling represents the probability of finding an agent in a certain position

Figure 1(b) shows the estimated probability density function (PDF) of the positions of all agents during the entire experiment. The color scale shows that VDVs tend to be on the edges of the arena. In particular, there is a high probability of finding agents around the opening.

We recorded the experiment using a GoPro camera in a zenith position with a time resolution 1/30 s. The recording time was ≈ 60 min.

3 Results

We studied the evolution of the density of particles ρ around the opening. For this, we measured the distance of the κ nearest-neighbor d_κ and estimated the density as

$$\rho \propto \frac{\kappa - 1}{d_\kappa^2} \ . \tag{1}$$

Note that the missing proportionality constant will have no effect on the results obtained. In this analysis, we used $\kappa = 3$. The analysis of the influence of κ on the emerging stylized facts is beyond the scope of this work but, we found results to be robustly present in the interval $\kappa \in [2, 4]$. Following the analogy put forward by Parisi et al. [2], we calculated 'returns' of the time series Y over a window of size j as

$$R_Y(t_i, j) = Y(t_{i+j}) - Y(t_i) \ , \tag{2}$$

where, for the VDV system, Y is the density of particles given by Eq. 1. Figure 2(a) shows the time evolution of the density of VDVs for 100 seconds. Data reveal alternating periods of high and low density. The temporal evolution of R_ρ presents periods of high variability clustering in time as can be seen in Fig. 2(b). Finally, the probability distribution funcion of R_ρ reveals a peaky distribution with fat tails.

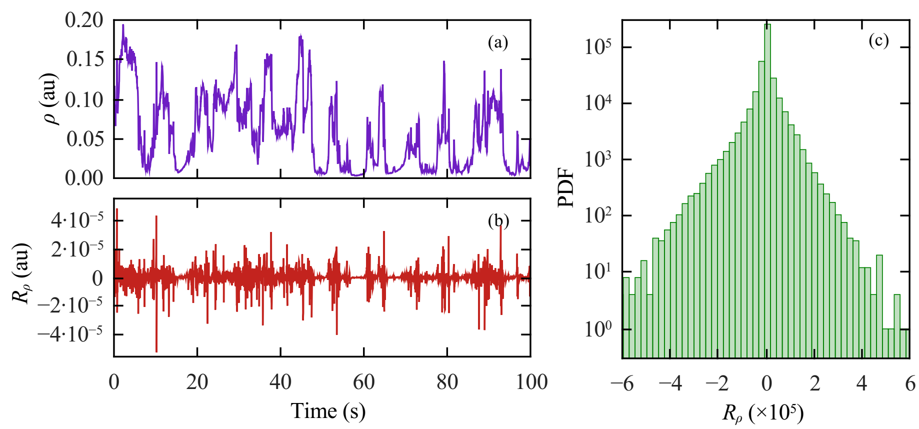

Fig. 2. Temporal evolution of (a) the density of particles and (b) the returns R_ρ for an arbitrary time window of 100 seconds. (c) Probability distribution function of R_ρ for the entire experiment.

Using the above definition of returns, we computed different statistical properties and compared them to the financial stylized facts. Specifically, we took the time series of the Bitcoin crypto-currency expressed in US dollars (BTC) ranging from 2012/12/31 to 2018/06/30 at a sampling rate of 1 hour [11]. In this case, Y is the log-price of BTC.

We first analyzed the correlations of several functions of R_Y. The sample correlation function is defined as

$$C(k) = \mathrm{corr}(R_Y(t_{i+k}), R_Y(t_i)) \ , \tag{3}$$

where k is the time lag. Figure 3(a) shows results for the VDV system revealing the absence of linear autocorrelation for times larger than a few lags. The BTC system presents the same behavior as can be seen in Fig. 3(b).

The volatility clustering of the returns observed in Fig. 2(b) was quantified by the autocorrelation function of an arbitrary power of the absolute returns as

$$C_\alpha(k) = \mathrm{corr}(|R_Y(t_{i+k})|^\alpha, |R_Y(t_i)|^\alpha) \ . \tag{4}$$

In contrast to $C(k)$, Fig. 3(a) shows that the autocorrelations of $C_1(k)$ and $C_2(k)$ for the VDV system present long memory properties characterized by a long correlation time in accordance with the BTC system (Fig. 3(b)) [12, 13].

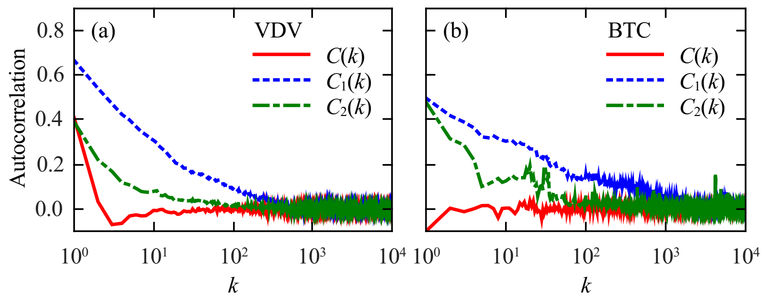

Fig. 3. (a) Autocorrelation functions for the variation of density R_ρ, $|R_\rho|$ and $|R_\rho|^2$ of the VDV system. (b) Same results for the BTC system

Distributions of financial returns are characterized by being non-Gaussian and fat tailed [1, 14]. Figure 4(a) shows the complementary cumulative distribution function (CCDF) of the standardized absolute returns $|R^*_Y|$, defined as $\frac{|R_Y|}{\langle |R^*_Y| \rangle}$, in comparison to that of the nearest Gaussian CCDF. Note that, as j increases, the CCDFs of returns converge to a Gaussian distribution. Figure 4(b) shows that the BTC system presents the same behavior.

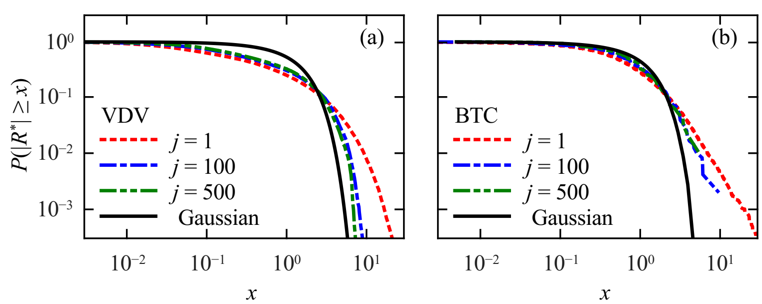

Fig. 4. CCDF for normalized absolute returns series R^*_ρ (a) and R^*_{BTC} (b) for various time windows j. Solid black lines stand for the CCDF of the nearest Gaussian distribution

The properties of the central part of the distribution were also investigated. For this, we estimated the probability of zero returns ($P(R_Y = 0)$) for different steps j as shown in Fig.5(a) and (b) for the VDV and BTC systems, respectively. We found that $P(R_Y = 0)$ decays as a power law function $j^{-\delta}$ in accordance with results obtained for the S&P500 index [15].

Finally, we studied the long-range dependence of the time series of absolute returns by means of the detrended function analysis (DFA) [16, 17]. Figure 6 shows the root-mean-square deviation from the trend $F(n)$ as a function of n.

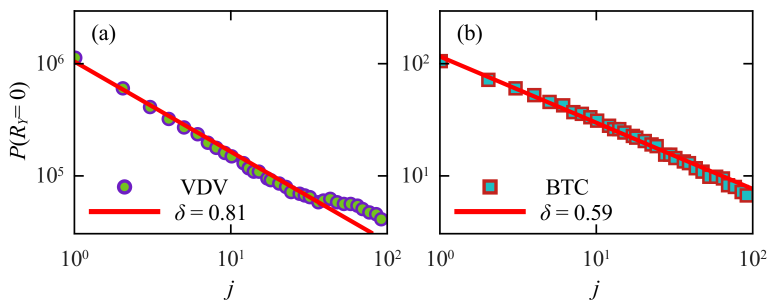

Fig. 5. Estimated probability of zero return as a function of the time step j for the VDV (a) and BTC. Red solid lines stand for power law fits

Both, the VDV (a) and BTC (b) systems, present a linear relationship in log-log scale. We fit power law functions and estimated Hurst exponents $H = 0.82$ and $H = 0.81$ for the VDV and BTC systems, respectively. Again, both systems are in accordance showing that periods of positive trends are followed by periods with the same trends.

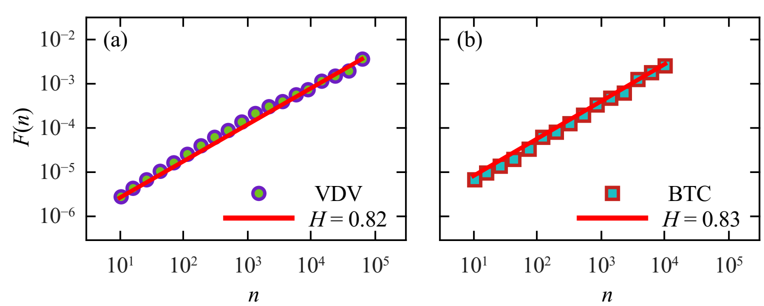

Fig. 6. Detrended function analysis for the VDV (a) and BTC (b) systems. Solid lines stand for power law fits

4 Conclusions

We studied the temporal evolution of the density of particles around the opening and make a statistical comparison with the price evolution of bitcoin. In this work, we report several analogies between these systems by measuring various statistical properties of the return of the market price and its correspondence of the mechanical system. Interestingly, for the chosen experimental conditions, we found that the time series of density shares several of the stylized facts found in the Bitcoin crypto-currency price. Although the reasons for these similarities remain unknown, this simple mechanical toy-model can be used to get insights into more complex financial systems.

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