Evaluation of the status of rotary machines by time causal Information Theory quantifiers

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A B S T R A C T

In this paper several causal Information Theory quantifiers, i.e. Shannon entropy, statistical complexity and Fisher information using the Bandt and Pompe permutation probability distribution, measure are applied to describe the behavior of a rotating machine. An experiment was conducted where a rotating machine runs balanced and then, after a misalignment, runs unbalanced. All the causal Information Theory quantifiers applied are capable to distinguish between both states and grasp the corresponding transition between them.

1. Introduction

Normally, the status of rotary machines cannot be evaluated directly. Instead, vibration signals generated from rotary machines have often been measured and then analyzed in order to evaluate the machine working status [1]. The application of signal processing in an earlier stage can prevent unexpected failures avoiding possible liabilities, such as cost of human life, production disrupt and other financial losses, enabling an increase of up to 30% in the productivity of the operation [2]. Friction, strikes, clearance between mobile parts, fractures in the bench or broken fasteners may occur during the lifetime of the machine [3]. These are all sources of non-stationary and non-linear vibrations and, therefore, the traditional study of the vibrations through linear methods may not be effective at all to detect dynamic changes on the signal.

Interesting results have been reported by different research groups in recent years, in the characterization of different dynamical systems following the temporal evolution of the signal dimensionality (associated with the measurement of the

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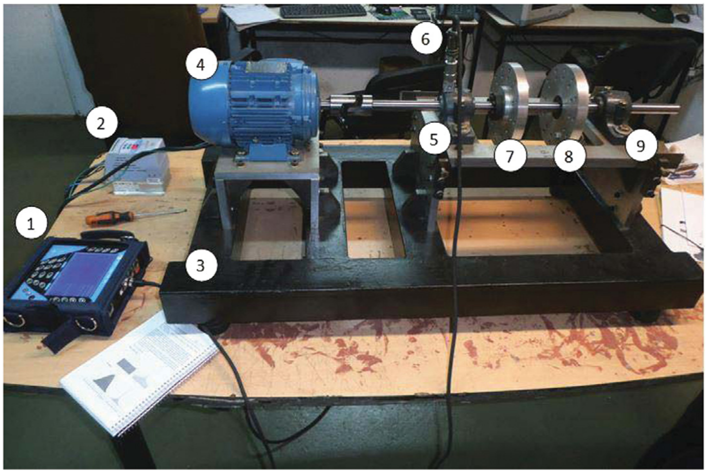

Fig. 1. Vibration Bench used in the present work. Main components: (1) Vibration Data Logger. (2) Variable Frequency Drive (VFD). (3) Bench. (4) Electric motor. (5) Bearing Housing 1. (6) Accelerometer. (7) Balancing Disc 1. (8) Balancing Disc 2. (9) Bearing Housing 2.

correlation dimension, D_2) and the chaoticness degree (through the largest Lyapunov exponent, Λ_{max}) (see i.e. Refs. [4–7]). It is important to note that the stationarity of the time series is a basic requirement in order to apply nonlinear dynamic metric tools (Chaos Theory) to experimental data. Unfortunately, as mentioned before, this is not the case of signals generated by rotary machines in general. Even more, in order to obtain reliable results, long time recordings with very low noise are required for the evaluation of D_2 and Λ_{max} (defined as asymptotic properties of the attractor).

In more recent works, the use of quantifiers based on Information Theory has led to interesting results regarding the characteristics of nonlinear chaotic dynamics, improving the understanding of their associated time series. In particular, the combination of the statistical complexity [8–11] and the normalized Shannon entropy, allows a good distinction between stochastic and chaotic dynamics when incorporating time causal information via the Bandt and Pompe methodology (the permutation probability distribution function (PDF) associated to a time series) [12,13]. In particular, symbolic time-series analysis methods that discretize the raw time series into a corresponding sequence of symbols, have the potential of analyzing nonlinear data efficiently with low sensitivity to noise and do not require the stationary condition. In Ref. [1] permutation entropy is used in order to characterize the bearing working status under different operating conditions. Permutation entropy (the Shannon entropy evaluated with Bandt and Pompe PDF) is, as it will be explained in Section 3, a global measure of the time series behaviour. In the present paper another global measure, the permutation statistical complexity, and a local measure, the permutation Fisher information, are proposed.

In order to characterize the transition of rotary machine, the use of time causal Information Theory quantifiers, Normalized Shannon Entropy, Statistical Complexity and Fisher Information Measure, are proposed. As for the organization of present work, Section 2 describes the experimental setup and present the time series to be analyzed. Sections 3 and 4 describe the Information Theory quantifiers, and the Bandt–Pompe methodology for the probability distribution function respectively. Finally, Section 5 presents our results and discussions.

2. Experimental setup and time series data

Rotary machine is formed by a number of components which interact with each other when the machine is in operating condition. The vibration signal measure obtained at this state is complex due to strong interference noises. The resulting vibration becomes more complex when there are faults on a component [1]. A series of experiments have been done, in the Centro de Materiales (CEMAT), at the Instituto Tecnológico de Buenos Aires, ITBA. A bench specifically designed (see Fig. 1), to simulate different failure conditions in a rotary machine was used for vibration measurements.

An electric motor marked by **(4)** in Fig. 1 is connected to a shaft through a universal joint. The shaft is mounted on two bearings, **(5)** and **(9)** in Fig. 1 and, has coupled two perforated discs **(7)** and **(8)** in Fig. 1 that can be positioned to generate unbalance eccentric loads. The bearing housings are bolted to a base that can be misaligned. A VFD allows to regulate motor speed. The measuring device **(1)** is a DSP Logger MX 300 with an accelerometer, **(6)** with 10 mV/g of sensitivity. It is configured to measure the displacement, velocity and acceleration and its coupled to a bearing housing by a headless bolt. The motor was started and the samples values were taken at 20 Hz as the motor speed.

Acceleration measurements (time series) were first made under no-load condition then with shaft misalignment and finally changing one of the bearings for another one with pitting in the outer race. Machine samples have been taken at different working conditions:

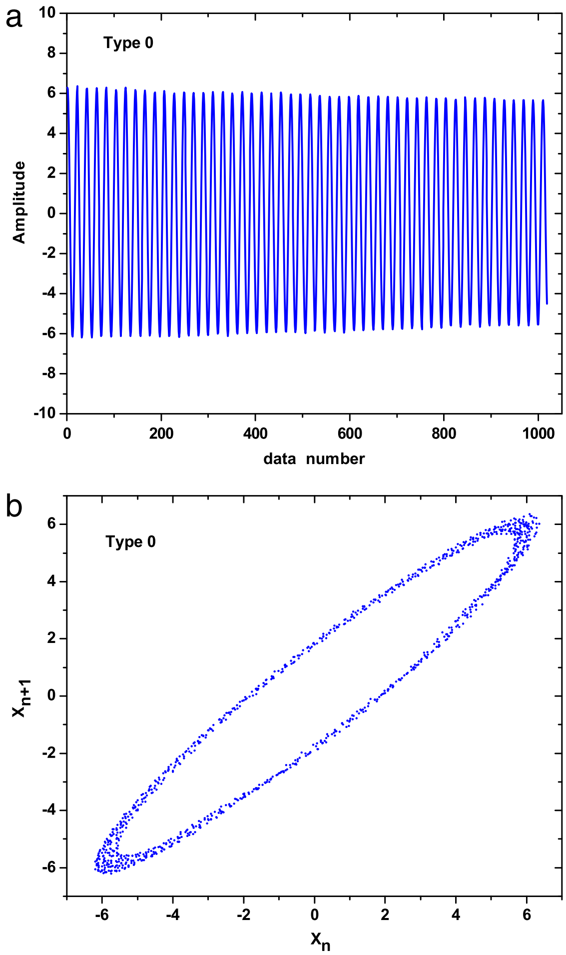

Fig. 2. Time series corresponding to balanced rotary machine. Condition denoted as Type 0.

- No-load condition: the machine is working in a normal condition. The shaft is balanced. We labeled this condition as Type 0. A typical signal for this condition is shown in Fig. 2(a). The dynamics in this condition is quite regular with almost periodic amplitude variations (see Fig. 2(b)).
- Unbalanced shaft: the motor shaft drives two aluminum disks where it is possible to add masses in order to displace the center of gravity from the system's rotation axis. Consequently, it produces vibrations on the machine. Masses (bolts and nuts) have been added to the disc 1 and/or 2 to create this unbalanced state. A typical signal for this condition is shown in Fig. 3.

  The proposed unbalanced states effectuated at the rotary machine are:

- Type 1: two additional masses, located at the external holed-rings coaxially.
- Type 2: two additional masses, located at the external holed-rings, with a 90° phase displacement.
- Type 3: one additional mass located at the external holed-ring. Only in one of the discs.
- Type 4: one additional mass located at the internal holed-ring. Only in one of the discs.
- Type 5: like Type 3 but the mass located in a hole at a 90° phase displacement.
- Type 6: like Type 4 but the mass located in a hole at a 90° phase displacement.

All the assays have been performed at a frequency of 20 Hz and the accelerometer has always been placed in the first bearing, the nearest to the motor. For each condition, time series of 1024 points have been taken. Six different signals (time series) were generated by connecting signal of Type 0 with the remaining signals Type 1–6, in this way typical different abrupt fault can be obtained. We display in Fig. 4 the six different time series to be analyzed. In the figures, the dots vertical line, represents the fault instant (data 1024) from which the rotary machine dynamics change from balanced to non-balanced.

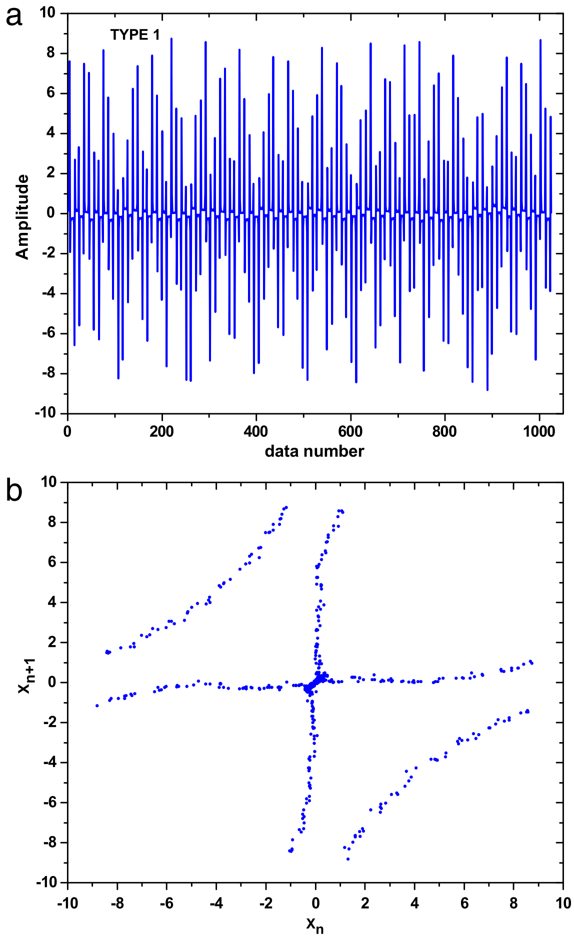

Fig. 3. Time series corresponding to unbalanced rotary machine. Condition denoted as Type 1.

3. Information theory quantifiers

Given a continuous probability distribution function (PDF) $f(x)$ with $x \in \Delta \subset \mathbb{R}$ and $\int_\Delta f(x)\, dx = 1$, its associated *Shannon Entropy* S [14] is

$$S[f] = -\int_\Delta f\ \ln(f)\, dx, \tag{1}$$

a measure of "global character" that it is not too sensitive to strong changes in the distribution taking place on a small-sized region. Such is not the case with *Fisher's Information Measure* (FIM) $\mathcal{F}$ [15,16], which constitutes a measure of the gradient content of the distribution $f(x)$, thus being quite sensitive even to tiny localized perturbations. It reads

$$\mathcal{F}[f] = \int_\Delta \frac{1}{f(x)} \left[\frac{df(x)}{dx}\right]^2 dx = 4\int_\Delta \left[\frac{d\psi(x)}{dx}\right]^2. \tag{2}$$

The Fisher's Information Measure can be variously interpreted as a measure of the ability to estimate a parameter, as the amount of information that can be extracted from a set of measurements, and also as a measure of the state of disorder of a system or phenomenon [16]. The gradient operator significantly influences the contribution of minute local f-variations to FIM's value. Accordingly, this quantifier is called a "local" one [16].

Let now $P = \{p_i;\ i = 1, \ldots, N\}$, with $\sum_{i=1}^{N} p_i = 1$, be a discrete probability distribution (PDF), with N the number of possible states of the system under study. The concomitant problem of information-loss due to discretization has been thoroughly studied and, in particular, it entails the loss of FIM's shift-invariance, which is of no importance for our present purposes [17,18]. In the discrete case, we define a "normalized" Shannon entropy ($0 \le \mathcal{H} \le 1$) as

$$\mathcal{H}[P] = S[P]/S_{max} = \left\{-\sum_{i=1}^{N} p_i\ \ln(p_i)\right\} \Big/ S_{max}, \tag{3}$$

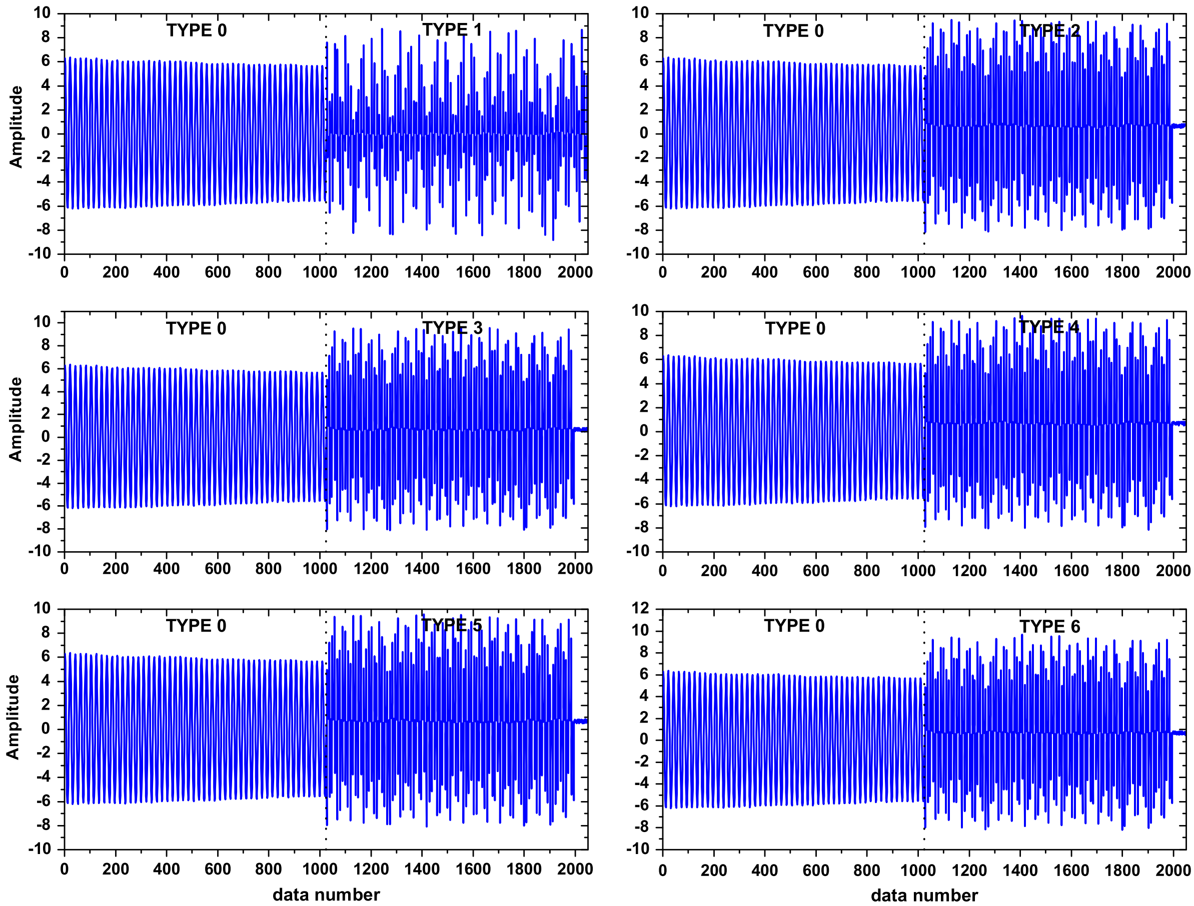

Fig. 4. Time series corresponding to rotary machine with different abrupt fault. The fault time is indicated by vertical dots line (data number 1024), and divide the signal on balanced to non-balanced behavior.

where the denominator $S_{max} = S[P_e] = \ln N$ is that attained by a uniform probability distribution $P_e = \{p_i = 1/N, \ \forall i = 1, \ldots, N\}$. For the FIM we take the expression in terms of real probability amplitudes as starting point, then a discrete normalized FIM convenient for our present purposes, is given by

$$\mathcal{F}[P] = F_0 \sum_{i=1}^{N-1} [(p_{i+1})^{1/2} - (p_i)^{1/2}]^2. \tag{4}$$

It has been extensively discussed that this discretization is the best behaved in a discrete environment [19]. Here the normalization constant F_0 reads

$$F_0 = \begin{cases} 1 & \text{if } p_{i^*} = 1 \text{ for } i^* = 1 \text{ or } i^* = N \text{ and } p_i = 0 \ \forall i \neq i^* \\ 1/2 & \text{otherwise.} \end{cases} \tag{5}$$

It is well known, however, that the ordinal structures present in a process is not quantified by randomness measures, and consequently, measures of statistical or structural complexity are necessary for a better understanding (characterization) of the system dynamics represented by their time series [20]. The opposite extremes of perfect order (i.e., a periodic sequence) and maximal randomness (i.e., a fair coin toss) are very simple to describe, because they do not have any structure. The complexity should be zero in these cases. At a given distance from these extremes, a wide range of possible ordinal structures exists. This behavior is quantified by the *Statistical Complexity* measure $\mathcal{C}$ along with critical details of the underlying dynamical processes in the data set.

Based on the seminal notion advanced by López-Ruiz et al. [21], this statistical complexity measure is defined through the functional product form:

$$\mathcal{C}[P] = \mathcal{Q}_J[P, P_e] \cdot \mathcal{H}[P] \tag{6}$$

of the normalized Shannon entropy $\mathcal{H}$ and the disequilibrium $\mathcal{Q}_J$ defined in terms of the Jensen–Shannon divergence. That is,

$$\mathcal{Q}_J[P, P_e] = Q_0 \, \mathcal{J}[P, P_e] \tag{7}$$

with:

$$\mathcal{J}[P, P_e] = S[(P + P_e)/2] - S[P]/2 - S[P_e]/2 \tag{8}$$

being $\mathcal{J}$ Jensen–Shannon divergence, and Q_0 a normalization constant equal to the inverse of the maximum possible value of $\mathcal{J}[P, P_e]$, (so $0 \le \mathcal{Q}_J \le 1$). This value is obtained when one of the components of the PDF, P, say p_m, is equal to one and the remaining p_j are equal to zero. Note that the above introduced statistical complexity depends on two different probability distributions, the one associated with the system under analysis, P, and the uniform distribution, P_e. Furthermore, it was shown that for a given value of $\mathcal{H}$, the range of possible $\mathcal{C}$ values varies between a minimum $\mathcal{C}_{min}$ and a maximum $\mathcal{C}_{max}$, restricting the possible values of the statistical complexity in a given entropy-complexity plane [22]. Thus, it is clear that important additional information related to the correlational structure between the components of the physical system is provided by evaluating the statistical complexity measure.

4. The Bandt–Pompe approach to the PDF determination

The study and characterization of time series $\mathcal{X}(t)$ by recourse to Information Theory tools assume that the underlying PDF is given *a priori*. In contrast, part of the concomitant analysis involves extracting the PDF from the data, and there is no univocal procedure with which everyone agrees. Bandt and Pompe (BP) introduced a successful methodology for the evaluation of the PDF associated with scalar time series data using a symbolization technique [12]. For a didactic description of the approach, as well as its main biomedical and econophysics applications, see Ref. [13].

The pertinent symbolic data are: (i) created by ranking the values of the series; and (ii) defined by reordering the embedded data in ascending order, which is tantamount to a phase space reconstruction with embedding dimension (pattern length) D and time lag τ. In this way, it is possible to quantify the diversity of the ordering symbols (patterns) derived from a scalar time series.

Note that the appropriate symbol sequence arises naturally from the time series, and no model-based assumptions are needed. In fact, the necessary "partitions" are devised by comparing the order of neighboring relative values rather than by apportioning amplitudes according to different levels. This technique, as opposed to most of those in current practice, takes into account the temporal structure of the time series generated by the physical process under study. This feature allows us to uncover important details concerning the ordinal structure of the time series [18,8,11] and can also yield information about temporal correlation [23,24].

It is clear that this type of analysis of a time series entails losing some details of the original series' amplitude information. Nevertheless, by just referring to the series' intrinsic structure, a meaningful difficulty reduction has indeed been achieved by Bandt and Pompe with regard to the description of complex systems. The symbolic representation of time series by recourse to a comparison of consecutive ($\tau = 1$) or nonconsecutive ($\tau > 1$) values allows for an accurate empirical reconstruction of the underlying phase-space, even in the presence of weak (observational and dynamic) noise [12]. Furthermore, the ordinal patterns associated with the PDF are invariant with respect to nonlinear monotonous transformations. Accordingly, nonlinear drifts or scaling artificially introduced by a measurement device will not modify the estimation of quantifiers, a nice property if one deals with experimental data (see, e.g., Ref. [25]). These advantages make the Bandt and Pompe methodology more convenient than conventional methods based on range partitioning (i.e., PDF based on histograms).

Additional advantages of the method reside in: (i) its simplicity; few parameters are needed: the pattern length/embedding dimension D and the embedding delay τ; and (ii) the extremely fast nature of the pertinent calculation process [26]. The BP methodology can be applied not only to time series representative of low dimensional dynamical systems, but also to any type of time series (regular, chaotic, noisy or reality based). In fact, the existence of an attractor in the D-dimensional phase space is not assumed. The only condition for the applicability of the Bandt–Pompe methodology is a very weak stationary assumption (that is, for $k \le D$, the probability for $x_t < x_{t+k}$ should not depend on t [12]).

To use the Bandt and Pompe [12] methodology for evaluating the PDF, P, associated with the time series (dynamical system) under study, one starts by considering partitions of the pertinent D-dimensional space that will hopefully "reveal" relevant details of the ordinal structure of a given one-dimensional time series $\mathcal{X}(t) = \{x_t; t = 1, \ldots, M\}$ with embedding dimension $D > 1$ ($D \in \mathbb{N}$) and embedding time delay τ ($\tau \in \mathbb{N}$). The "ordinal patterns" of order (length) D generated by

$$(s) \mapsto \left(x_{s-(D-1)\tau},\ x_{s-(D-2)\tau},\ \ldots,\ x_{s-\tau},\ x_s \right), \tag{9}$$

which assigns to each time s the D-dimensional vector of values at times $s, s - \tau, \ldots, s - (D - 1)\tau$. Clearly, the greater the D-value, the more information on the past is incorporated into our vectors. By "ordinal pattern" related to the time (s), the permutation $\pi = (r_0, r_1, \ldots, r_{D-1})$ of $[0, 1, \ldots, D - 1]$ defined by

$$x_{s-r_{D-1}\tau} \le x_{s-r_{D-2}\tau} \le \cdots \le x_{s-r_1\tau} \le x_{s-r_0\tau} \tag{10}$$

is used. In order to get a unique result, we set $r_i < r_{i-1}$ if $x_{s-r_i} = x_{s-r_{i-1}}$. This is justified if the values of x_t have a continuous distribution, so that equal values are very unusual. Thus, for all the $D!$ possible permutations π of order D, their associated relative frequencies can be naturally computed by the number of times this particular order sequence is found in the time series divided by the total number of sequences,

$$p(\pi) = \frac{\sharp\{s | s \le M - D + 1;\ (s), \texttt{has type}\ \pi\}}{M - D + 1}. \tag{11}$$

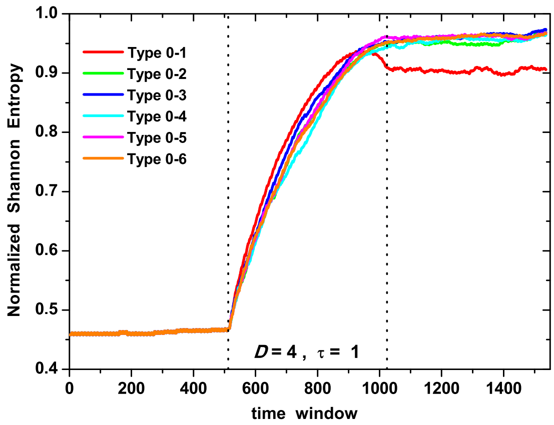

Fig. 5. Time evolution of normalized permutation Shannon entropy, $\mathcal{H}$, in overlapping sliding time windows of length $N = 512$ *data* and overlap $\delta = 1$ *data*, for the six signal displayed in Fig. 4. The Bandt–Pompe PDF was evaluated with pattern length of $D = 4$ and embedding time $\tau = 1$. The dot vertical lines represent the three different behaviors transitions time: pure Type 0, transitional, pure Type 1–6, respectively.

In this expression, the symbol $\sharp$ stands for "number".

Consequently, it is possible to quantify the diversity of the ordering symbols (patterns of length D) derived from a scalar time series, by evaluating the so-called permutation entropy (Shannon entropy), the permutation statistical complexity and Fisher permutation information measure. Of course, the embedding dimension D plays an important role in the evaluation of the appropriate probability distribution, because D determines the number of accessible states $D!$ and also conditions the minimum acceptable length $M \gg D!$ of the time series that one needs in order to work with reliable statistics [8].

Regarding the selection of the parameters, Bandt and Pompe suggested working with $4 \leq D \leq 6$ and specifically considered an embedding delay $\tau = 1$ in their cornerstone paper [12]. Nevertheless, it is clear that other values of τ could provide additional information. It has been recently shown that this parameter is strongly related, if it is relevant, to the intrinsic time scales of the system under analysis [27–29].

The Bandt and Pompe proposal for associating probability distributions to time series (of an underlying symbolic nature) constitutes a significant advance in the study of nonlinear dynamical systems [12]. The method provides univocal prescription for ordinary, global entropic quantifiers of the Shannon-kind.

The local sensitivity of the Fisher Information measure for discrete PDFs is reflected in the fact that the specific "*i*-ordering" of the discrete values p_i must be seriously taken into account in evaluating the sum in Eq. (4). The pertinent numerator can be regarded as a kind of "distance" between two contiguous probabilities. Thus, a different ordering of the pertinent summands would lead to a different Fisher information value. In fact, if a discrete PDF is given by $P = \{p_i, i = 1, \ldots, N\}$, we will have $N!$ possibilities for the i-ordering.

Which is the arrangement that one could regard as the "proper" ordering? In some cases, the answer is straightforward: the histogram-based PDF constitutes a conspicuous example. For such a procedure, one first divides the interval $[a, b]$ (with a and b the minimum and maximum amplitude values in the time series) into a finite number on non-overlapping subintervals (bins). Thus, the division procedure of the interval $[a, b]$ provides the natural order sequence for the evaluation of the PDF gradient involved in the Fisher information measure. In our current paper, for the Bandt–Pompe PDF the lexicographic ordering given by the algorithm of Lehmer [30] is chosen, amongst other possibilities, because it provides a better distinction of different dynamics in the Fisher–Shannon plane (see Ref. [17,18]).

5. Results and discussion

The six signals displayed in Fig. 4 were analyzed using the Information Theory based quantifiers: normalized Shannon entropy, statistical complexity and Fisher information measure, all the three quantifiers evaluated with Bandt–Pompe PDF. In particular, the study of interest is the behavioral transition between the different dynamics, from balanced to non-balanced. In order to do that, the above quantifiers are evaluated in overlapping sliding time windows of length $N = 512$ *data* and overlap $\delta = 1$ *data*. For the Bandt and Pompe PDF evaluation a pattern length (embedding dimension) of $D = 4$ and embedding time $\tau = 1$ are considered. The three quantifiers time evolution, normalized Shannon entropy, statistical complexity and Fisher information are given in Figs. 5 to 7 respectively. Each point represents the quantifier value in the corresponding time window. From these figures, one can see three different distinguishable behavioral zones. The first one which can be associated with a balance behavior (pure signal Type 0), which corresponds to time window 1–512, a transitional behavioral zone between windows 513–1024, and a third zone, corresponding to non-balance behavior (pure signal Type 1–6) between windows 1025–1537.

As in Ref. [1], normalized permutation Shannon entropy, $\mathcal{H}$, could effectively detect changes in the dynamics of a rotary machine status. The same is valid for the other Information Theory quantifiers: the permutation statistical complexity, $\mathcal{C}$

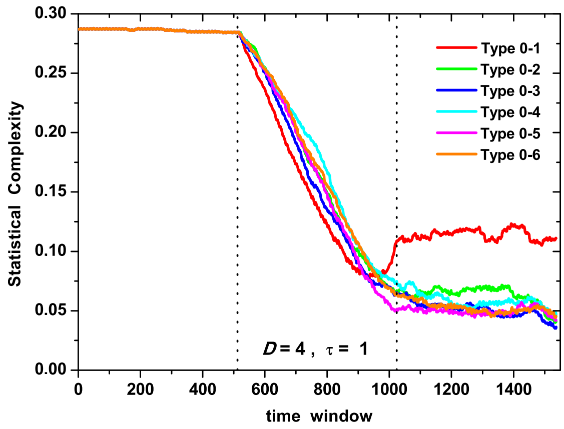

Fig. 6. Same as Fig. 5 for permutation statistical complexity, $\mathcal{C}$.

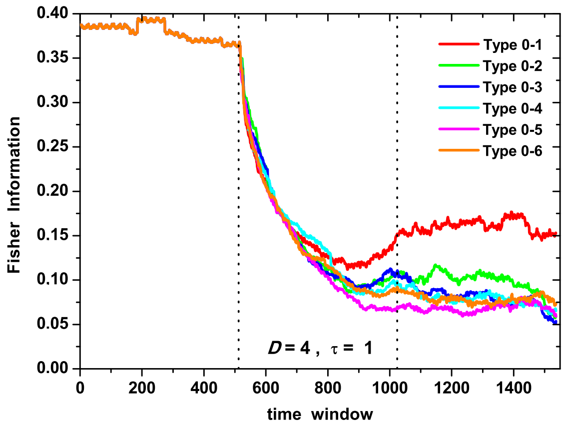

Fig. 7. Same as Fig. 5 for permutation Fisher information measure, $\mathcal{F}$.

Table 1
Mean and standard deviation values for: normalized Shannon entropy, $\mathcal{H}$; statistical complexity, $\mathcal{C}$; and Fisher information, $\mathcal{F}$, evaluated with Bandt–Pompe PDF with $D = 4$ and $\tau = 1$, between time windows 1–512 for signal Type 0 and time windows 1025 and 1537 for signals Type 1 to 6.

	$\langle\mathcal{H}\rangle$	SD ($\mathcal{H}$)	$\langle\mathcal{C}\rangle$	SD ($\mathcal{C}$)	$\langle\mathcal{F}\rangle$	SD ($\mathcal{F}$)
Type 0	0.46212	0.00307	0.28622	0.00116	0.37958	0.00941
Type 1	0.90437	0.00341	0.11385	0.00431	0.16037	0.00753
Type 2	0.95265	0.00521	0.06359	0.00709	0.09828	0.01232
Type 3	0.96190	0.00464	0.05119	0.00617	0.08094	0.01133
Type 4	0.95495	0.00501	0.05880	0.00607	0.07898	0.00720
Type 5	0.96145	0.00225	0.04979	0.00298	0.06860	0.00420
Type 6	0.96002	0.00388	0.05278	0.00452	0.07883	0.00541

and, the permutation Fisher information measure, $\mathcal{F}$. All the analyzed signals present a pseudoperiodic or noisy periodic behavior. In particular, this behavior is more evident in the case of balance function (signal Type 0, see Fig. 2) which is characterized by low normalized permutation Shannon entropy, medium permutation statistical complexity and medium permutation Fisher information values. Behavior compatible with medium degree of order in the signal, which is clear if one looks at Fig. 2(b). In opposition, unbalanced functional behavior (signals Type 1–6) is characterized by high $\mathcal{H}$, low $\mathcal{C}$ and low $\mathcal{F}$ values. That means, a disordered and noise behavior can be associated for all unbalanced behaviors. Table 1 presents the mean values and standard deviation for the three quantifiers over the 1–512 windows for Type 0 signal and, over the 1025–1537 time window for Type 1–6 signals. Note that the behavior of three quantifiers, normalized permutation Shannon entropy, permutation statistical complexity and permutation Fisher information, over this time intervals is quite stable, as can see from Figs. 5–7, and also from Table 1. From the above figures, it can be observed that the transitional region (between window 513–1024) is characterized by a fast increasing values on $\mathcal{H}$ and decreasing $\mathcal{C}$ and $\mathcal{F}$, behavior compatible with increasing increment in the degree of disorder in the signals induced by the un-balance. Taking into account these facts, one can propose a simple method for the detection of behavioral change in motor function monitoring, making a flag mark when the some of the based Information Theory quantifiers in a time window present a value greater than or equal to $3\sigma^2$ (with σ^2 the corresponding standard deviation) of its mean value in the balanced behavior.

It is interesting to note that the inclusion of the Fisher information measure for the characterization of the rotary machine paves the way to study local changes in the behavior of the signal, it could be an important factor in the detection of the reason of the failure, discriminating local changes from global ones in the dynamics, however it needs further research in that way.

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