Fault tolerance analysis of a multirotor with 6DOF

Juan Giribet, Claudio Pose and Ignacio Mas

***Abstract*— Several works have addressed the issue of obtaining a six degree of freedom (6DOF) capable multirotor. However, the capability of keeping full control in the case of a total failure in one of the rotors has not yet been analyzed.**

In this work, it is proven that eight is the minimum number of rotors necessary to achive 6DOF control of a multirotor, in the case of a total failure in one of them.

Also, an octorotor configuration with 6DOF control capabilities in a failure situation is proposed and analyzed through numerical simulations, comparing its flight performance in a nominal and a failure case.

I. Introduction

During the last years, unmanned multirotor vehicles have become very popular, both in recreational applications and in professional ones. These vehicles are usually composed of a number of rotor-propeller sets that provide thrust, uniformly distributed over a main frame, with a battery that provides power, and inertial and position sensors for navigation. The simplicity of the mechanical design, the minimum necessary number of moving parts, and the always decreasing cost of the electronics, transformed these vehicles in a simple, economic platform for diverse applications, such as professional filming, dangerous areas exploration, and academic research. With its increasing use, an increasing number of accidents came along, turning fault tolerance into a critical issue.

Several works have addressed issues of fault tolerance, usually focusing on actuator failures, as sensor failures may be overcome using hardware redundancy, as they are both cheap and lightweight. In this line of work, different approaches were taken, considering flying losing control over some degrees of freedom [1], reconfiguration of the platform to support some (but not all) rotor failures [2], in-air dynamic reconfiguration [3], or using over-actuated platforms [4].

On the other hand, omnidirectional multirotor vehicles have also been studied. In contraposition to the traditional multirotor vehicles, omnidirectional multirotor can decouple the translational and rotational dynamics. For instance, they can translate without the need of tilting the vehicle, which can be useful for aerial filming and 3D mapping. It could be argued that it would be enough if a gimbal is installed on the vehicle to stabilize the sensors. However, if the sensors are too heavy, the additional gimbal weight will dramatically reduce the time of flight.

Several designs have been proposed for obtaining fully-actuated omnidirectional multirotor vehicles, see for instance [5], [6], [7], [8], and references therein. There is a difference between the aforementioned designs, as some of them cannot exert force in every direction. The downward force is given by the vehicle weight, this is the case of [7] and many other similar designs. In some situations, it would be necessary to be capable of exerting a downward force larger than the vehicle's weight, and rotors pointing downwards would need to be included. However, in many cases, for instance for mapping applications, it is enough with the limit imposed by the vehicle's weight, assumption that is adopted in this work. For these cases, it is well known (see for instance [9], [10] and references therein) that an hexacopter with the rotors tilted inward a small angle is capable of achieving 6DOF, assuming that the pitch and roll angles are not larger than certain values that guarantee that the vertical force exerted is enough to compensate for vehicle weight, i.e., the vehicle does not fly upside down, which is a standard assumption for most multirotor applications.

On the other hand, in [11] it has been proven that a tilted rotor hexacopter is capable of 4DOF, even if one rotor fails. It was shown that it is capable of full-attitude and altitude control. The proof presented in [11] is based on a result that can be found in [12], where a fault tolerant thruster-based attitude control system was studied for spacecraft. Based on these results, recently an omnidirectional multirotor vehicle was proposed [13].

The idea of this work is to study a 6DOF fault tolerant control system for a multirotor vehicle. The main contribution is to extend the results presented in [11], and study the minimum number of rotors needed to achieve a 6DOF fault tolerant multirotor vehicle.

II. Notation and Background

Given $X \in \mathbb{R}^{n\times m}$, $N(X)$ denotes its kernel, $R(X)$ its range, $rg(X)$ its rank and X^T its transpose. The i-th element of vector $v \in \mathbb{R}^m$ is denoted v_i. Given a subspace $\mathcal{S} \subseteq \mathbb{R}^n$, $dim(\mathcal{S})$ denotes the dimension of $\mathcal{S}$.

A vector $u \in \mathbb{R}^n$, is called positive (strictly positive), and denoted $u \geq 0$ ($u > 0$), if its components are positive, i.e., $u_1 \geq 0, ..., u_n \geq 0$ ($u_1 > 0, ..., u_n > 0$). The notation $c\theta = \cos(\theta)$ and $s\theta = \sin(\theta)$ is used in order to make some equations look more compact. The following theorem, which has been proven in [12], will be useful to analyze the existence of positive solutions of linear equations.

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Theorem 2.1: Let $A \in \mathbb{R}^{m\times n}$, the following conditions are equivalent.

1) For each $q \in \mathbb{R}^m$ there exists $p \geq 0$ such that $q = Ap$.
2) Matrix A has full rank m and there exists $w \in N(A)$ with strictly positive components, i.e., $w > 0$.

Suppose a vehicle with $n \in \mathbb{N}$ rotors capable of generating only positive forces $p \in \mathbb{R}^n$ such that $p \geq 0$. Also assume that $B_q \in \mathbb{R}^{3\times n}$ is a matrix relating these forces with the torque acting on the vehicle. More specifically, if $q \in \mathbb{R}^3$ is the torque produced by forces p then

$$q = B_q p. \tag{1}$$

Theorem 2.1 establishes that it is possible to exert torque in every direction if and only if B_q is full-rank and there is a strictly positive vector in $N(B_q)$. When working with an omnidirectional vehicle, not only it is desirable to reach any torque but also exert force in every direction. Then, if $q \in \mathbb{R}^3$ is the desired torque and $f \in \mathbb{R}^3$ is the desired force, it is possible to reach these torque and forces with the vehicle if and only if there exists a positive solution of

$$\begin{bmatrix} q \\ f \end{bmatrix} = \begin{bmatrix} B_q \\ B_f \end{bmatrix} p,$$

where $B_f \in \mathbb{R}^{3\times n}$ is the matrix relating the force of each rotor and the resultant force on the vehicle. Which is equivalent to studying $N(A)$, with $A = \begin{bmatrix} B_q \\ B_f \end{bmatrix}$.

When working with multirotor vehicles, it is not necessary to exert negative forces in the vertical direction; because the weight of the vehicle is enough to make it go down. For omnidirectional multirotor vehicles, the rotors must be capable of reaching torque and lateral forces in any direction, and positive vertical forces. It must be studied if it is possible with positive rotor forces $p \in \mathbb{R}^n$ to achieve arbitrary torque $q \in \mathbb{R}^3$ and lateral forces $f_\ell \in \mathbb{R}^2$

$$\begin{bmatrix} q \\ f_\ell \end{bmatrix} = \begin{bmatrix} B_q \\ B_{f_{red}} \end{bmatrix} p,$$

where $B_{f_{red}}$ is a reduced version of matrix B_f, which only contemplates the lateral forces.

III. Rotors failure model

Without failures, the relation between forces exerted by each rotor $p \in \mathbb{R}^n$ ($n > 5$) and, torque $q \in \mathbb{R}^3$ and lateral force $f_\ell \in \mathbb{R}^2$ is given by matrix

$$A_{red} = \begin{bmatrix} B_q \\ B_{f_{red}} \end{bmatrix} \in \mathbb{R}^{5\times n}. \tag{2}$$

In the case of a total failure in a rotor i, the same analysis can be performed on the matrix A_{red}, by eliminating the $i-th$ column corresponding to the rotor in failure (denoted $A^i_{red} \in \mathbb{R}^{5\times(n-1)}$). Then, with a failure in rotor i, the vehicle is capable of achieving an arbitrary torque q and lateral forces f_ℓ if and only if, there exist $p \in \mathbb{R}^{n-1}$, such that $p \geq 0$ and

$$\begin{bmatrix} q \\ f_\ell \end{bmatrix} = A^i_{red}\, p,$$

or equivalently, if A^i_{red} is full rank and there exist $w \in N(A^i_{red})$ such that $w > 0$.

Observe that, there exists a strictly positive vector $w \in \mathbb{R}^{n-1}$ such that $w \in N(A^i_{red})$ if and only if there exists $\tilde{w} \in \mathbb{R}^n$ such that $\tilde{w}_i = 0$ and $\tilde{w}_j > 0$, for every $j \neq i$.

The idea is to find the minimum number of rotors such that the vehicle is capable of 6DOF even in the presence of a failure in one of its rotors. The following lemma will be useful for such purpose.

Lemma 3.1: Suppose that $x, y, z \in \mathbb{R}^n$ are vectors such that $x_i = 0$ and $x_j > 0$, for every $j \neq i$, $y_k = 0$ for some $k \neq i$ and $y_j > 0$, for every $j \neq k$, and $z_l = 0$ for some $l \neq i, l \neq k$ and $z_j > 0$, for every $j \neq l$. Then x, y, z are linearly independent vectors.

Proof: Suppose that $x, y, z \in \mathbb{R}^n$ satisfy the hypothesis, and let $N = \begin{bmatrix} x & y & z \end{bmatrix}$ be the matrix containing vectors $x, y, z \in \mathbb{R}^n$ as columns. Observe that, there exists a row-permutation matrix $U \in \mathbb{R}^{n\times n}$ such that

$$M = UN = \begin{bmatrix} 0 & m_{12} & m_{13} \\ m_{21} & 0 & m_{23} \\ m_{31} & m_{32} & 0 \\ m_{41} & m_{42} & m_{43} \\ \vdots & \vdots & \vdots \\ m_{n1} & m_{n2} & m_{n3} \end{bmatrix},$$

with $m_{ij} > 0$. Since

$$det \begin{bmatrix} 0 & m_{12} & m_{13} \\ m_{21} & 0 & m_{23} \\ m_{31} & m_{32} & 0 \end{bmatrix} > 0,$$

it follows that the columns of M are linearly independent vectors and then also are columns of N because U is invertible. ■

Notice that, if the vehicle is capable of reaching any torque and lateral force with motor 1 in failure, then matrix A^i_{red} must satisfy $rg(A^i_{red}) = 5$, and a vector $x \in \mathbb{R}^n$ (with $x_1 = 0, x_i > 0$ for every $i = 2, ..., n$) must satisfy $x \in N(A_{red})$. But also vectors $y \in \mathbb{R}^n$ (with $y_2 = 0, y_i > 0$ for every $i = 1, 3, ..., n$) and $z \in \mathbb{R}^n$ (with $z_3 = 0, z_i > 0$ for every $i = 1, 2, 4, ..., n$) must belong to $N(A_{red})$. Then, by Lemma 3.1 vectors $x, y, z \in \mathbb{R}^n$ are linearly independent, then the following result can be established.

Lemma 3.2: Let $A_{red} \in \mathbb{R}^{5\times n}$ as in equation (2). If this matrix represents a vehicle capable of reaching torque and lateral forces in any direction, even in presence of a failure in one of its rotors, then $dim(N(A_{red})) \geq 3$ and for every $i = 1, ..., n$, $rg(A^i_{red}) = 5$.

Based in Lemma 3.2 it can be inferred that, a fault tolerant multirotor capable of exerting torque and lateral forces in any direction must have at least $n = 8$ rotors, because $rg(A_{red}) = 5$ and $N(A_{red})$ contains, at least, three linear independent vectors.

$$A(\gamma) = \begin{bmatrix} \tilde{k}_t[& -c\gamma_1 & \frac{\sqrt{2}}{2}\frac{1+\alpha(\gamma_2)}{\alpha(\gamma_2)}c\gamma_2 & \frac{c\gamma_3}{\alpha(\gamma_3)} & \frac{\sqrt{2}}{2}\frac{1-\alpha(\gamma_4)}{\alpha(\gamma_4)}c\gamma_4 & c\gamma_5 & -\frac{\sqrt{2}}{2}\frac{1+\alpha(\gamma_6)}{\alpha(\gamma_6)}c\gamma_6 & \frac{c\gamma_7}{\alpha(\gamma_7)} & -\frac{\sqrt{2}}{2}\frac{1-\alpha(\gamma_8)}{\alpha(\gamma_8)}c\gamma_8] \\ \tilde{k}_t[& \frac{c\gamma_1}{\alpha(\gamma_1)} & \frac{\sqrt{2}}{2}\frac{1-\alpha(\gamma_2)}{\alpha(\gamma_2)}c\gamma_2 & c\gamma_3 & -\frac{\sqrt{2}}{2}\frac{1+\alpha(\gamma_4)}{\alpha(\gamma_4)}c\gamma_4 & -\frac{c\gamma_5}{\alpha(\gamma_5)} & -\frac{\sqrt{2}}{2}\frac{1-\alpha(\gamma_6)}{\alpha(\gamma_6)}c\gamma_6 & -c\gamma_7 & \frac{\sqrt{2}}{2}\frac{1+\alpha(\gamma_8)}{\alpha(\gamma_8)}c\gamma_8] \\ \tilde{k}_t[& s\gamma_1 & -s\gamma_2 & s\gamma_3 & -s\gamma_4 & s\gamma_5 & -s\gamma_6 & s\gamma_7 & -s\gamma_8] \\ [& c\gamma_1 & \frac{\sqrt{2}}{2}c\gamma_2 & 0 & -\frac{\sqrt{2}}{2}c\gamma_4 & -c\gamma_5 & -\frac{\sqrt{2}}{2}c\gamma_6 & 0 & \frac{\sqrt{2}}{2}c\gamma_8] \\ [& 0 & -\frac{\sqrt{2}}{2}c\gamma_2 & -c\gamma_3 & -\frac{\sqrt{2}}{2}c\gamma_4 & 0 & \frac{\sqrt{2}}{2}c\gamma_6 & c\gamma_7 & \frac{\sqrt{2}}{2}c\gamma_8] \\ [& s\gamma_1 & s\gamma_2 & s\gamma_3 & s\gamma_4 & s\gamma_5 & s\gamma_6 & s\gamma_7 & s\gamma_8] \end{bmatrix} \quad (3)$$

The question is if $n = 8$ rotors are sufficient to satisfy this property. The answer is yes, and is stated in the following theorem.

Theorem 3.1: Number $n = 8$ is the minimum number of rotors for a vehicle to be able to exert torque and lateral forces in any direction, even in the presence of a failure in one of its rotors. Moreover, there exists a vehicle with 8 rotors capable of 6-DOF even with one rotor in complete failure.

The proof of this theorem consists in the design of a fault tolerant vehicle with 8 rotors capable of 6DOF. This design is given in Section V. But, first it is shown that tilting the rotors inward in an octorotor vehicle is not enough for a fault tolerant vehicle with 6DOF.

IV. Inward-tilted rotor configuration

Consider an octocopter which rotors are placed in a symmetric, co-planar distribution, as shown in Fig. 1, where the arms of the vehicle are tilted inwards at an angle γ_i for the i-th rotor, as in Fig. 2.

The first proposition analyzed for the orientation of the eight rotors, in the same line of work as [11], [14], was one where the rotors were tilted inwards at the same fixed angle γ. This solution proved to be effective to maintain 4DOF control in the event of a failure in a hexarotor, and it was investigated if performs in a similar way for 6DOF control in an octorotor with failure.

For this distribution, the relation between the rotor forces and the resulting torque and force acting on the vehicle is given by:

$$\begin{bmatrix} q \\ f \end{bmatrix} = A(\gamma) \cdot p, \qquad \text{with} \qquad p = \begin{bmatrix} p_1 \\ \vdots \\ p_8 \end{bmatrix}, \quad (4)$$

where the matrix $A(\gamma) \in \mathbb{R}^{6\times 8}$ is as shown in (3), for the case $\gamma_i = \gamma,\ i = 1, ..., 8$. The notation $\alpha(\theta) = \tilde{k}_t/(l\tan(\theta))$ is used, $l = \|d_i\|$ and $\tilde{k}_t = k_t/k_f$.

As stated in Section II, to check the full controllability of a vehicle capable of exerting any torque and lateral forces, it suffices to check for the full rank of A_{red}, and the existence of a positive vector in the kernel of the matrix.

$$\begin{bmatrix} q \\ f_\ell \end{bmatrix} = = A_{red}(\gamma) \cdot p, \qquad \text{with} \qquad p = \begin{bmatrix} p_1 \\ \vdots \\ p_8 \end{bmatrix}. \quad (5)$$

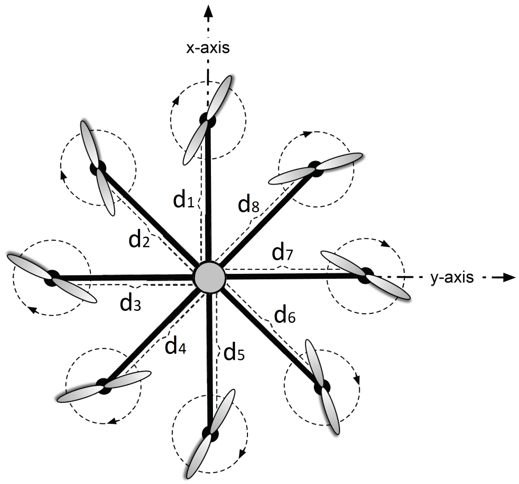

Fig. 1. Top view of the co-planar octorotor.

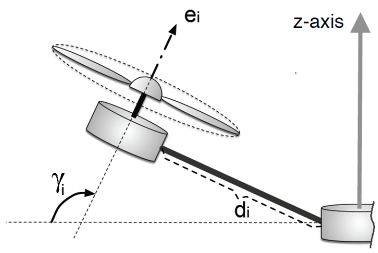

Fig. 2. Side view of the co-planar octorotor and the inward-tilt angle γ.

For the case without failures, $A_{red} \in \mathbb{R}^{5\times 8}$, $rank(A_{red}) = 5$, and the null space is generated by:

$$N(A_{red}) = span \left\{ \begin{array}{c} [1 \ \ 1 \ \ 1 \ \ 1 \ \ 1 \ \ 1 \ \ 1 \ \ 1], \\ [1 \ \ 1 \ \ 0 \ \ 0 \ \ 1 \ \ 1 \ \ 0 \ \ 0], \\ [1 \ \ 0 \ \ 0 \ \ 1 \ \ 1 \ \ 0 \ \ 0 \ \ 1] \end{array} \right\}.$$

Then, the vehicle without failures is controllable in 6DOF. The vehicle is also able to increase the vertical force to ascend, independently of the torques and of the lateral forces, and, while it cannot produce force downwards, its wight allows to go down. In the case of a total failure in a rotor i, the same analysis can be performed on the matrix A^i_{red}. For example, for the rotor number 2 in failure, the rank of $A^2_{red} \in \mathbb{R}^{5\times 7}$ is still 5, but its null space is generated by:

$$N(A^2_{red}) = span \left\{ \begin{array}{c} [-1 \quad 1 \quad 0 \quad -1 \quad 0 \quad 1 \quad 0], \\ [1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 0 \quad 1] \end{array} \right\}.$$

In this case, the vehicle can not achieve torque and lateral forces in any direction, due to the lack of a strictly positive vector in the kernel of A^2_{red}. The situation is the same if the failure occurs in any of the other rotors, meaning that the proposed configuration is not fault tolerant for any case of a total failure in one of the rotors.

V. Proposed Solution

To find a feasible distribution for the rotors, numerical simulations were carried out for a variety of possible solutions, analyzing the maximum torque and force that could be achieved by each of them in case of a total failure in any of the rotors. In cases where the maneuverability of the vehicle depends on which is the rotor that presents a failure, the worst case situation is assumed (that which restricts the most the $[q, f]$ set achievable).

The proposed solution in this work is to use a coplanar, circular distribution as in Fig. 1, with a different inward/outward tilt angle for each rotor. From rotor 1 to 8, the proposed tilt γ_i are:

$$\begin{aligned} &\gamma_1 = \pi/2, \gamma_2 = \delta, \gamma_3 = \pi - \beta, \gamma_4 = \delta, \\ &\gamma_5 = \pi/2, \gamma_6 = \pi - \delta, \gamma_7 = \beta, \gamma_8 = \pi - \delta \end{aligned} \tag{6}$$

where the tilt angle $\beta = acos(2cos(\delta)cos(\pi/4)) = acos(\sqrt{2}cos(\delta))$. Then, only δ needs to be defined, leaving a one variable problem. The particular case for $\delta = \pi/2$ is not considered, as it corresponds to a standard octocopter distribution that is not able to produce forces in the *XY* plane.

The proposed solution shows some sort of symmetry, but the performance of the vehicle with a failure present will still depend on which rotor is failing.

The matrix $A(\delta)$ in this case is also represented by equation (3), replacing in each column the angle γ_i with the proposed configuration.

It is easy to prove that both $A_{red}(\delta) \in \mathbb{R}^{5\times 8}$ and $A^i_{red}(\delta) \in \mathbb{R}^{5\times 7}$ for $i = 1, ..., 8$ have full rank and there exists a strictly positive vector in their null space. This results in a vehicle that has full control over its six degrees of freedom, both for the case without failure, and for the case of a failure in any rotor.

Numerical simulations are carried out for the proposed vehicle, using an estimated model of an octorotor, based on a 7Kg vehicle with a rotor-to-rotor distance of 80cm (then $l = 0.4m$), that holds rotor-propeller sets that provide a maximum of 2Kg thrust each. An estimated value of $\tilde{k}_t = 0.03$ is used, based in several bench tests with different types of motors, and plastic and carbon propellers.

In Fig. 3, the maximum torque achievable in the worst case direction is shown, for a variable value of the tilt angle δ, for a failure in each of the rotors from 1 to 8. Also, it is provided a comparison with the proposed vehicle and with an inward-tilted octorotor ($\gamma_i = \gamma,\ i = 1, ..., 8$), both in

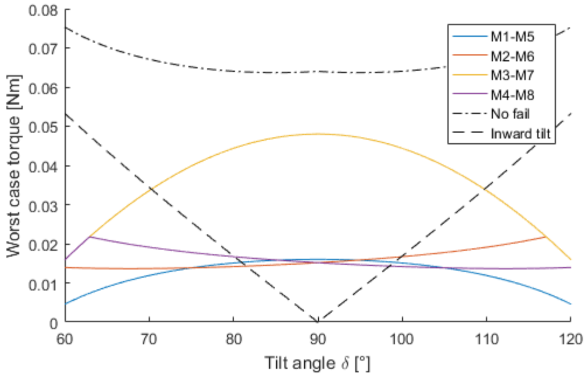

Fig. 3. Maximum torque achievable in any direction, for δ variable, for a failure in each rotor. Also, maximum torque for the proposed vehicle without failure, and for a symmetric inward tilted octorotor without failure.

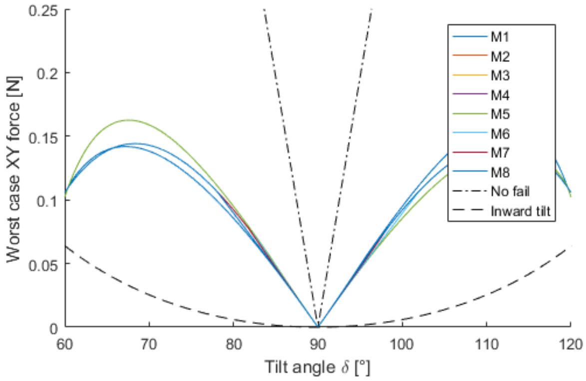

Fig. 4. Maximum force achievable in any direction, for δ variable, for a failure in each rotor. Also, maximum force for the proposed vehicle without failure, and for a symmetric inward tilted octorotor without failure.

a case with no failure. In a similar way, Fig. 4 shows the maximum force achievable in the *XY* plane.

In Fig. 3, while eight curves are plotted for the failure cases, only four are visible, due to the symmetry of the vehicle, resulting in the same curve for a failure in opposite rotors. This can also be observed in Fig. 4. As stated before, $\delta = \pi/2$ would result in a vehicle that only has control over 4 degrees of freedom (even in the no-failure case), which is indicated by the fact that Fig. 4 shows an achievable maximum force in the plane of zero.

Regarding the selection of δ, it is shown that using $\delta > \pi/2$ or $\delta < \pi/2$ produces the same result, due to symmetry. As δ is driven away from $\pi/2$, the maximum achievable torque in any direction decreases (supposing any rotor is equally probable to fail), while the maximum achievable force in the *XY* plane increases.

Something worth noticing is that the proposed vehicle performs better in case of a failure, compared to a symmetric inward-tilted vehicle without failure.

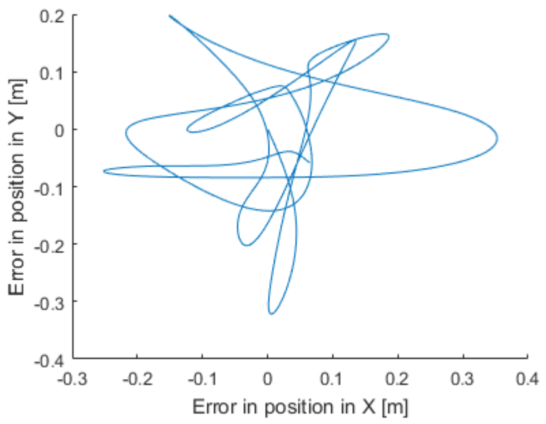

Fig. 5. Position error for the vehicle with no failures, subject to perturbations

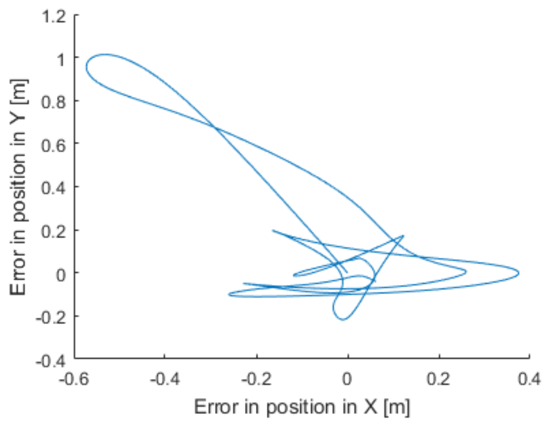

Fig. 6. Position error for the vehicle with a total failure in rotor 1, subject to perturbations

VI. Simulation Results

To evaluate the performance of the proposed vehicle, and to compare the no-failure and failure cases, a model was built using MATLAB Simulink. Two simulations are presented for each case, one considering position hold in a fixed point, and one considering a path to be traveled.

The simulation is based on a 6DOF motion block, and a control block that consists of three inner PID loops to control the attitude, and three outer PID loops that allow position control.

A tilt angle of $\delta = 110^o$ is chosen for the simulations, in order to have high lateral force in case of a failure, while not sacrificing too much torque. To analyze the failure case, a total failure is considered in rotor number 1. The reason to only consider this failure lies in the fact that the maximum torque achievable in any direction is the lowest among all the failure cases, which will provide the worst possible situation. The maximum achievable lateral force is similar for all failure cases, then the case of a failure in rotor number 1 can be analyzed without loss of generality.

In Figs. 5 and 6, the vehicle starts in a hovering state at 40m above ground, and holds position during a 60 second flight. In both figures, the error in position during flight is shown, for the case of the vehicle without failure, and for the case of a total failure in rotor number 1. The position control is done using PID loops over the pitch and roll angles, without controlling the lateral forces. A random perturbation profile was generated to simulate vibrations and possible winds that would affect the vehicle in the air. Both vehicles manage to hold position satisfactorily, with the failure case performing slightly worse, due to a more limited maneuverability.

In Fig. 7, a square path with a 1.8m side length is generated for the vehicle to follow. Also, a random perturbation profile (shown in Fig. 8) is generated and used for both cases to simulate winds and perturbations. The position control is done using PID loops over the lateral forces, and by setting the pitch and roll angles references to zero. While the same controllers are used for the no-failure and the failure cases, the maximum lateral force for the failure case is limited to $0.5N$, and for the no-failure case is limited to $0.8N$. This restriction is imposed to prevent the saturation of the rotors, but still allowing a good maneuverability. The vehicle without failures is able to generate much larger lateral forces, but was also limited to provide a reasonable comparison. Both vehicles manage to complete the path, again with the failure case performing slightly worse, as strong perturbations affect greatly the vehicle. In Fig. 9 the orientation of both vehicles during the path flight is shown. While both graphs present a similar shape (as the path and the perturbation profile is identical for both cases), the case without failure has a much better response, but the failure case still performs satisfactorily.

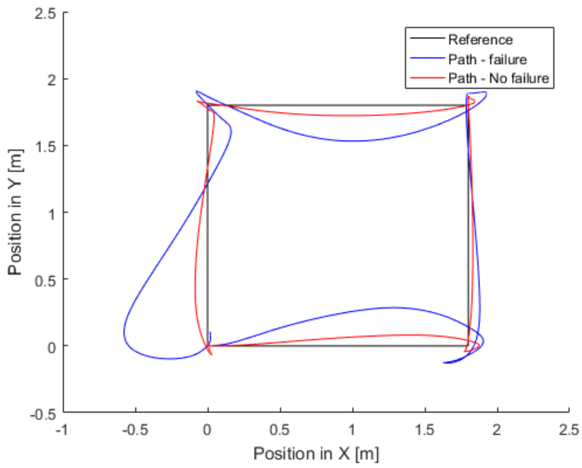

Fig. 7. Reference and simulated path for the vehicle with no failure, and with a total failure in rotor 1.

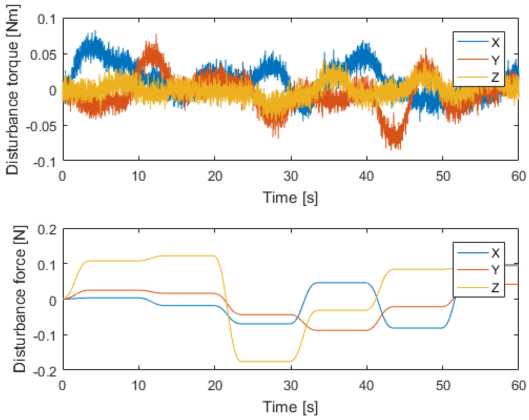

Fig. 8. Perturbation torques and forces during the path flight.

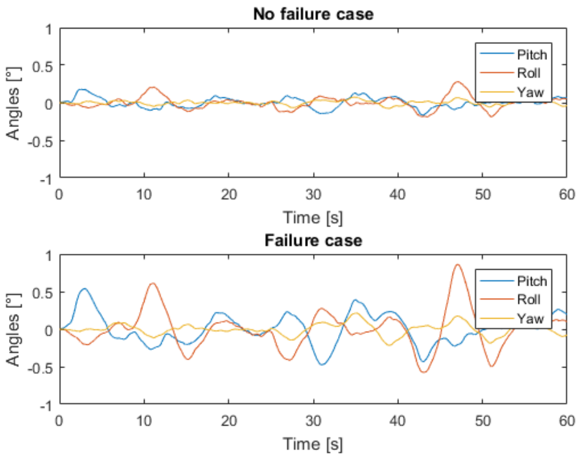

Fig. 9. Orientation of the vehicle without failure (top) and with failure (bottom) during the flight following the square path.

VII. Conclusions

In this work, it is shown that eight is the minimum number of rotors to have a vehicle controlled with 6DOF, even if one of its rotors fails.

The performance of the proposed design is evaluated with numerical simulations. A comparison is made between a vehicle following a trajectory, under nominal conditions and in the presence of a failure in one rotor. Although performance has degraded in the latter scenario, it still shows good overall performance.

For future work, it would be interesting to analyze the optimal configuration for the octorotor. In this work only two angles (β and δ) are taken as design parameters for rotor placement. This constraints are based on simplicity of the design, and not based on performance criteria. Also, the relation between these two angles is empirically obtained through numerical simulations.

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