Real time stable identification: A Nehari/SOS approach

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***Abstract*— Here we present an adaptive identification algorithm based on Second Order section (SOS) model structures. The procedure guarantees stable transfer functions whenever the actual physical plant is stable, due to an optimal Nehari approximation step performed analytically. The procedure is suitable to be implemented in real time applications. Some examples illustrate the proposed algorithm.**

I. INTRODUCTION

Adaptive identification algorithms have been used in the area of adaptive control systems for a very long time, both for feedback (FB) and/or feedforward (FF) approaches ([1], [2]). Usually for simplicity and computational speed in real time applications, parametric linear schemes have been implemented: RLS, NLMS, FXLMS, FULMS, as in the case of Active noise control ([3]), for example. Nevertheless the traditional assumptions in adaptive control: lack of perturbations or high frequency uncertain dynamics and minimum phase models, have generated at the end of the 80's an intense work in the area of robustness of adaptive laws ([2], [4], [5]). These have been extensively studied since then, and an excellent survey in this area can be found in [6].

Still then in adaptive identification, the stability of the resulting IIR model is generally not guaranteed, causing serious practical problems particularly in FF implementations. There are methods to convert IIR to FIR like the *Nehari shuffle* ([7]) and a recent LMI optimal version in [8], but the error is usually greater and requires a larger number of parameters in general. The use of IIR filters instead has the potential to decrease the identification error due to the fact that it includes the pole dynamics. In addition, this class of filters are in certain applications more efficient in modelling signals and require smaller model orders ([9]). Therefore an IIR filter that can guarantee a stable behavior and can be used in real time applications is a necessary tool in practical situations.

On the other hand, numerical problems also arise in real time applications, depending on the structural representation of the model. Take for example an 11th. order stable filter implemented with three different model structures: zero–pole (ZP), state space (SS) and transfer function (TF), the latter in terms of numerator and denominator coefficients, as follows:

$$\text{(ZP)} \qquad \prod_{i=1}^{m} \frac{(z_i z^{-1} - 1)}{(p_i z^{-1} - 1)}, \quad \text{(TF)} \ \frac{\sum_{i=0}^{m} z^{-i} b_i}{\sum_{i=0}^{m} z^{-i} a_i}$$

$$\text{(SS)} \qquad \begin{aligned} x_{k+1} &= Ax_k + Bu_k \\ y_k &= Cx_k + Du_k \end{aligned}$$

The complexity of each model is $\mathcal{O}(m^2)$ in the case of SS and $\mathcal{O}(m)$ in the other two cases, therefore from this point of view, the ZP and TF structures are more efficient. Nevertheless, it is a well known fact that the pole locations in the case of the TF structure, particularly in high order models, are significantly modified, even producing unstable poles ($|p_i| > 1$), as illustrated in Table I. On the other hand, it is easier to use the TF representation as the difference equation which implements the filter in real time, as follows:

$$\begin{aligned} y_k = \frac{1}{a_0} \qquad & [b_0\ u_k + \cdots + b_m\ u_{k-m} \\ & -a_1\ y_{k-1} - \cdots - a_m\ y_{k-m}] \end{aligned} \tag{1}$$

Therefore, the TF representation has advantages in terms of complexity and implementation, but serious disadvantages in terms of perturbations of pole locations, at least in high order models.

The solution to this problem is obtained by a series connection of Second Order Sections (SOS), which is an adequate way of implementing filters in real time. The SOS structure is numerically more efficient than the plain TF structure due to the fact that it has 2nd. order numerator and denominator, therefore preserving the original pole–zero locations. In addition, cascade-forms of SOS provide an attractive realization for adaptive IIR filters because the stability of the filter parametrization is easily monitored, and because filter pole locations are readily obtained from the adapted parameters with low computational cost ([10]).

In the previous example, the SOS' pole locations coincides with the ZP and SS structures. Furthermore it is still $\mathcal{O}(m)$ and each SOS can be implemented as a difference equation

ZP	SS	TF
0.0034	0.0034	0.0034
0.9975	0.9975	1.0295
0.9975	0.9975	1.0295
0.9949	0.9949	1.0128
0.9949	0.9949	1.0128
0.9607	0.9607	0.9924
0.9608	0.9608	0.9924
0.9802	0.9802	0.9646
0.9995	0.9995	0.9646
0.9995	0.9995	0.9434
0.9961	0.9961	0.9434

TABLE I

ABSOLUTE VALUE OF POLES OF A DISCRETE–TIME SYSTEM REPRESENTED IN ZERO–POLES (ZP), STATE–SPACE (SS) AND TRANSFER FUNCTION (TF).

connected in series with all other SOS', as follows:

$$\frac{Y(z)}{U(z)} = \prod_{i=1}^{m/2} \frac{z^{-2}b_2^i + z^{-1}b_1^i + b_0^i}{z^{-2}a_2^i + z^{-1}a_1^i + 1} \tag{2}$$

and each SOS is implemented as a 2nd. order difference equation:

$$y_k^i = b_0^i\, u_k^i + b_1^i\, u_{k-1}^i + b_2^i\, u_{k-2}^i - a_1^i\, y_{k-1}^i - a_2^i\, y_{k-2}^i$$

where $a_0^i = 1$ for simplicity.

There are many applications where a stable adaptive real time identification is needed, one of them being Active Noise Control (ANC) [11], [12], [3]. There, significant noise attenuation can be achieved through FB and/or FF controllers. In the first case, there are many well known limitations of the feedback loop that produces a poor performance. These performance limitations are mainly due to the non-minimum phase nature of the plant (see [13], [14], and also [15] and its revision in [16]), which in turn is derived from the time delay of sound propagation, e.g. acoustic tubes. Instead, a FF filter performs better because it is not restricted to the loop limitations. In this kind of application, the FF controller acts as a real time identifier of the acoustic noise signal received by the error microphone at the end of the tube, in order to cancel it at that point. Usually an adaptive identification scheme is used which can produce in many situations unstable behaviors. More details will be given in the application example at the end of this work. A complete experimental study of an hybrid – FF/FB controller applied to ANC in a tube can be found in [17].

As a consequence, a convergent adaptive identifier with guaranteed stable behavior and numerical robustness is very useful in these situations. Such an algorithm will be described in this work. Numerical stability is achieved by the use of SOS' and the stability of each section is guaranteed by a stable Nehari projection ([18]), which provides the nearest (optimal) stable model to a possibly unstable one. Due to the fact that the objective of this procedure is to implement it in real time situations, the Nehari projection is developed in analytical form.

The paper is organized as follows. Next section presents the background material: an analytic version of the Nehari stable projection and the adaptive identification procedure. Section III presents the main result of this work, followed by section IV with several examples which illustrate different aspects and limitations of this procedure. Final conclusions are drawn in section V.

II. PRELIMINARY RESULTS

A. Nehari's stable projection

Nehari's result is well known in the area of systems and control ([18]). It produces, both for discrete and continuous time systems, the optimal stable projection of a completely unstable system. Furthermore, the optimality implies that the resulting error is an all–pass filter with a gain corresponding to the highest Hankel singular value of the original system. It can be stated as follows:

Theorem 2.1: Given a completely unstable system $U(x)$, its optimal stable projection has the following solution:

$$\inf_{S\in\mathcal{H}_\infty} \|U(x) - S(x)\|_\infty = \|E(x)\|_\infty = \bar{\sigma}$$

where $\mathcal{H}_\infty$ is the Hardy space corresponding to causal transfer functions, $E(x)$ is an all–pass filter, $\bar{\sigma}$ is the larger Hankel singular value of $U^*(x) \in \mathcal{H}_\infty$ and variable x holds for either s or z, both complex variables corresponding to the Laplace and Z–transforms of continuous or discrete time systems, respectively.

Here we have computed analytically both, the stable projection and the corresponding error in the case of 2nd. and first order models. Due to the fact that the purpose is to achieve a real time fast implementation, we proceed, without loss of generality, by considering a discrete time model. Therefore, given a SOS with poles p_1 and p_2:

$$\begin{aligned} f(z) &= \frac{z^{-2}b_2 + z^{-1}b_1 + b_0}{z^{-2}a_2 + z^{-1}a_1 + 1} \\ &= \frac{\cdots}{(1+p_1\, z^{-1})(1+p_2\, z^{-1})} \\ p_1 &= \frac{a_1 - \sqrt{a_1^2 - 4a_2}}{2} \\ p_2 &= \frac{a_1 + \sqrt{a_1^2 - 4a_2}}{2} \end{aligned} \tag{3}$$

we first need to verify it's stability condition. There are 3 cases:

Stable poles: We do not perform any modification, i.e. $f_s(z) = f(z)$.

Stable–Unstable poles: We separate the SOS into it's stable and unstable parts, as follows:

$$f(z) = \underbrace{\frac{x_0 + x_1\, z^{-1}}{1 + p_1\, z^{-1}}}_{H_u} + \underbrace{\frac{1 + x_2\, z^{-1}}{1 + p_2\, z^{-1}}}_{H_s}$$

$$\begin{cases} x_0 = b_0 - 1 \\ x_1 = \frac{p_1^2 + x_o a_2 + b_2 - p_1 b_1}{\sqrt{a_1^2 - 4a_2}} \\ x_2 = \frac{p_2 b_1 - a_2 - x_o p_2^2 - b_2}{\sqrt{a_1^2 - 4a_2}} \end{cases}$$

where we have assumed that the unstable pole is $|p_1| > 1$. Similar results are derived by simply changing $p_1 \leftrightarrow p_2$ in the previous equations, if $|p_2| > 1$.

The optimal Nehari stable projection of the unstable term is X_{opt} with a constant approximation error over all frequencies $\bar{\sigma}$, producing a stable optimal approximation of the SOS $f_s(z)$:

$$\begin{aligned} X_{opt} &= \frac{x_0 - x_1 p_1}{1 - p_1^2} \\ \bar{\sigma} &= x_1 - p_1 X_{opt} \\ f_s(z) &= X_{opt} + H_s(z) \\ &= \frac{(X_{opt} + 1) + (x_2 + p_2 X_{opt}) z^{-1}}{1 + p_2 z^{-1}} \end{aligned}$$

Unstable poles: Here both poles are unstable, i.e. $|p_1| > 1$ and $|p_2| > 1$. The optimal Nehari stable projection of $f(z)$ is:

$$f_s(z) = \frac{n_0 + n_1 z^{-1}}{1 + d_1 z^{-1}}$$

$$\begin{cases} n_0 = b_0 - a_2 d_1 \bar{\sigma} \\ n_1 = b_1 - a_1 b_0 - a_2 \bar{\sigma} - d_1 \left[\bar{\sigma} a_1 (1 - a_2) - b_0\right] \\ d_1 = \frac{a_2 b_1 - a_1 a_2 b_0 + \bar{\sigma}(1 - a_2^2)}{a_2[\bar{\sigma} a_1 (1 - a_2) - b_0] + b_2} \end{cases}$$

and the constant approximation error over all frequencies is the solution to the following quadratic equation:

$$\alpha \bar{\sigma}^2 + \beta \bar{\sigma} + \gamma = 0$$

$$\begin{cases} \alpha = 1 - a_1^2 (a_2 - 1)^2 + a_2^2 (a_2^2 - 2) \\ \beta = a_1 (a_2 - 1)^2 (b_0 + b_2) - b_1 + a_2 b_1 (1 + a_2 - a_2^2) \\ \gamma = a_1 a_2 b_0 b_1 - (a_2 b_0)^2 - a_2 b_1^2 - b_0 b_2 a_1^2 \\ \qquad + 2 a_2 b_0 b_2 + a_1 b_1 b_2 - b_2^2 \end{cases}$$

The choice of $\bar{\sigma}$ corresponds to the stable case, i.e. $|d_1| < 1$.

B. Adaptive identification procedure

Without loss of generality we could use any robust adaptive identification procedure, due to the fact that our algorithm relies on the SOS structure and the optimal Nehari stable approximation (in closed form). Here we have decided to test the algorithm using the well known Switching σ-modification. ([2]). This adaptive identification algorithm, can be stated as follows [5]:

Given the i–th SOS as in equation (3):

$$f_i(z) = \frac{z^{-2} b_2^i + z^{-1} b_1^i + b_0^i}{z^{-2} a_2^i + z^{-1} a_1^i + 1}$$

define the parameters and regressors as follows:

$$\theta_k^i = \begin{bmatrix} b_0^i \\ b_1^i \\ b_2^i \\ a_1^i \\ a_2^i \end{bmatrix}, \quad r_k^i = \begin{bmatrix} u_k^i \\ u_{k-1}^i \\ u_{k-2}^i \\ -y_{k-1}^i \\ -y_{k-2}^i \end{bmatrix}$$

Assume that we know a bound M^i of the norm of the true parameter Θ^i, *i.e.* $\|\Theta^i\| \leq M^i$, where $\|\cdot\|$ is the Euclidean norm of a vector. Then the algorithm which produces the i–th SOS model y_{mk}^i at time $t = t_k$, and the parameter update is:

$$\begin{aligned} y_{mk}^i &= [r_k^i]^T \theta_{k-1}^i && (4) \\ K_k^i &= \frac{c\, r_k^i}{\alpha + \left\|r_k^i\right\|^2} && (5) \\ \epsilon_k^i &= K_k^i \left[y_k^i - y_{mk}^i\right], && (6) \\ \theta_k^i &= (1 - \sigma_k^i)\theta_{k-1}^i + \epsilon_k^i, && (7) \end{aligned}$$

where (c, α) are the convergence gain and *forgetting* factor parameters, both chosen by the designer with $0 < c < 1$ and

$$\sigma_k^i = \begin{cases} \sigma_0 & \text{if} \quad \|\theta_k^i\| \geq 2M^i \\ 0 & \text{if} \quad \|\theta_k^i\| < 2M^i, \end{cases}$$

and where $0 < \sigma_0 < (1 - c)/2$.

Next, and due to the fact that this algorithm cannot assure the stability of the SOS, we apply the Nehari projection algorithm described above, and obtain the estimates θ_k^i of the i–th SOS.

III. Main algorithm

A. I/O propagation

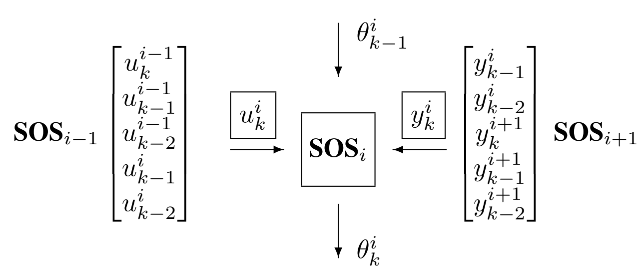

Fig. 1. Inputs and outputs propagation scheme.

The algorithm identifies each SOS separately connecting the results altogether. To this end, the inputs of the i–th SOS need to be defined as the outputs propagated from the $(i -$

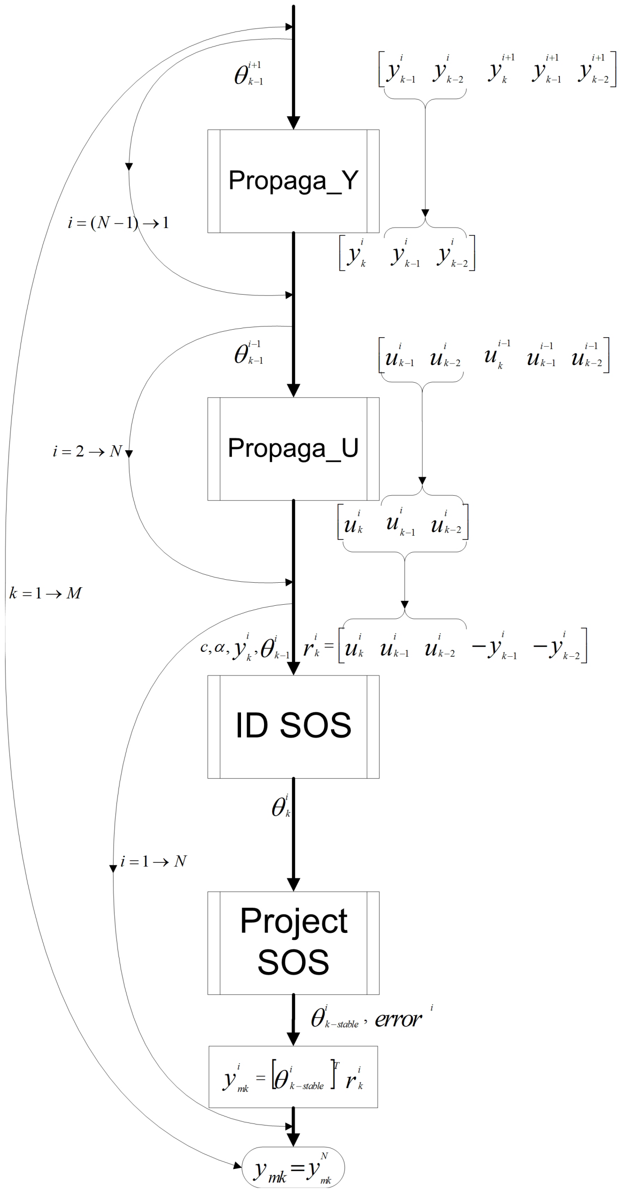

Fig. 2. Algorithm flow diagram.

1)–th SOS. Similarly, the outputs from the i–th SOS are (inversely) propagated from the $(i+1)$–th SOS, as follows:

$$
\begin{aligned}
u_k^i &= \left[u_k^{i-1} \quad u_{k-1}^{i-1} \quad u_{k-2}^{i-1} \quad -u_{k-1}^{i} \quad -u_{k-2}^{i}\right]\theta_{k-1}^{i-1} \\
y_k^i &= \frac{1}{b_0^{i+1}}\left\{\left[0 \quad -y_{k-1}^{i} \quad -y_{k-2}^{i} \quad y_{k-1}^{i+1} \quad y_{k-2}^{i+1}\right]\theta_{k-1}^{i+1}\right. \\
&\qquad \left. +y_k^{i+1}\right\}
\end{aligned}
$$

Here, the first input coincides with the physical input $u_k^1 = u_k$ and the same for the last (N–th) output $y_k^N = y_k$. In the propagation of the input signal u_k^i, the updated value of the parameters θ_k^{i-1} can also be used. The propagation scheme has been represented in figure 1 and is part of the main algorithm in figure 2 as the blocks `Propaga_Y` and `Propaga_U`.

B. Algorithm

The complete flow diagram is illustrated in figure 2. The external loop goes from time t_k to t_M and produces the model output y_{mk} at time t_k. The propagation scheme has been explained previously and produces the inputs and outputs for each SOS, here denoted generically as the i–th SOS. These will be part of the regressor vector r_k^i used in the next step. The identification (`ID SOS`) has been explained in section II-B and produces the i–th SOS updated parameter vector θ_k^i at time t_k. As explained previously, at this stage any convergent identification algorithm can be applied. Next, and depending on the stability of the SOS due to the values of the parameters θ_k^i, the projection procedure, explained in section II-A, is applied at stage `Project SOS` in figure 2. It produces a stable set of parameters $\theta_{k-stable}^i$ that will be used to produce the model output y_{mk}^i, according to equation (4). An error term $y_k^i - y_{mk}^i$ is also generated to keep track of the convergence of the whole procedure. The number of SOS sections is arbitrary, and the total order of the system's model can be odd. In that case, the remaining first order section can be easily projected in case it gets unstable at some point in the identification procedure. This has been explained in section II-A, as a particular case when only one of the poles of a SOS section is unstable.

C. Analysis

The effects of both, the propagation scheme (`Propaga_Y`, `Propaga_U`) and the Nehari projection (`Project SOS`), on each SOS can be represented by a modeling error process, and the system can be described by the following difference equation:

$$
\begin{aligned}
y_k^i &= b_0^i\, u_k^i + b_1^i\, u_{k-1}^i + b_2^i\, u_{k-2}^i - a_1^i\, y_{k-1}^i - a_2^i\, y_{k-2}^i \\
&+ \Delta_k^i
\end{aligned}
$$

with modeling errors Δ_k^i that satisfy

$$
|\Delta_k^i| \le \delta_k^i\, \|r_k^i\|_2 + \mu_k^i, \quad \delta_k^i \in \mathcal{L}_\infty, \quad \mu_k^i \in \mathcal{L}_\infty
$$

The introduction of the term $-\sigma_k^i\theta_{k-1}^i$ in the adaptive identification algorithm (a modification in fact of the well known gradient algorithm) guarantees the boundedness of θ_k^i and that $\left\{\left[y_k^i - y_{mk}^i\right]^2/(\alpha + \left\|r_k^i\right\|^2)\right\} \in \mathcal{L}_2$. This effect, that is local for each SOS, produces a robust, with respect to the initial estimates of the parameters, identification algorithm for the whole cascade.

It must be pointed out that if any other robust identification algorithm is used instead of the Switching σ-modification, similar results will be obtained.

The proof of these statements will be published elsewhere.

IV. Examples

Example 4.1: This example is focused on the stabilizing effect of the Nehari projection. A 32nd. order stable ($|p_i| <$

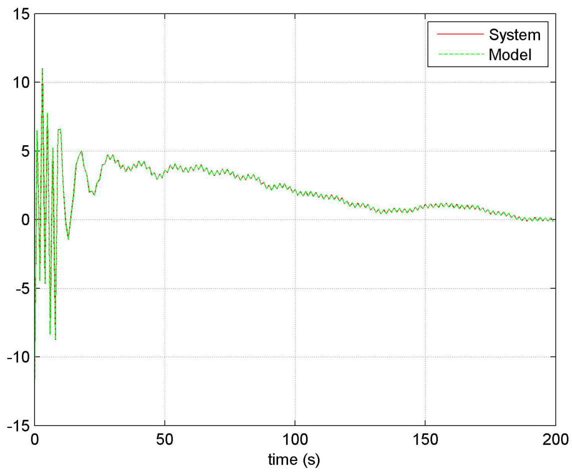

Fig. 3. 32nd. order model approximation using NLMS and the Projection algorithm.

1) and non–minimum phase model has been identified. The poles ($\mathbf{p}$) and zeros ($\mathbf{z}$) of this system are presented next.

$$\mathbf{p} = \begin{bmatrix} -0.9969 \\ 0.3817 \\ 0.6905 \pm \jmath 0.4938 \\ 0.8193 \pm \jmath 0.5465 \\ 0.8984 \pm \jmath 0.4235 \\ 0.9100 \pm \jmath 0.4076 \\ 0.9193 \pm \jmath 0.3379 \\ 0.9536 \pm \jmath 0.2942 \\ 0.9619 \pm \jmath 0.2637 \\ 0.9697 \pm \jmath 0.2349 \\ 0.9069 \pm \jmath 0.1479 \\ 0.9647 \pm \jmath 0.1750 \\ 0.9888 \pm \jmath 0.0084 \\ 0.9890 \pm \jmath 0.0783 \\ 0.9849 \pm \jmath 0.1418 \\ 0.9785 \pm \jmath 0.1292 \\ 0.9918 \pm \jmath 0.1179 \end{bmatrix} \qquad \mathbf{z} = \begin{bmatrix} 0.0016 \\ 0.0268 \\ 1.1514 \\ 0.9875 \\ 0.8210 \pm \jmath 0.5672 \\ 0.9035 \pm \jmath 0.4243 \\ 0.9115 \pm \jmath 0.4074 \\ 0.9211 \pm \jmath 0.3616 \\ 0.9390 \pm \jmath 0.3224 \\ 0.9525 \pm \jmath 0.2928 \\ 0.9605 \pm \jmath 0.2621 \\ 0.9687 \pm \jmath 0.2344 \\ 0.9716 \pm \jmath 0.2041 \\ 0.9773 \pm \jmath 0.1669 \\ 0.9951 \pm \jmath 0.0756 \\ 0.9864 \pm \jmath 0.1430 \\ 0.9861 \pm \jmath 0.1185 \\ 0.9915 \pm \jmath 0.1178 \end{bmatrix}$$

The input to the system is an impulse and two different identification procedures have been implemented, the NLMS ([1]) and the Projection algorithm ([2]). The model and system outputs are illustrated both in figure 3, due to the fact that the results are similar. In both cases, the tracking of the system is very good, hence the identification error is bounded and very small. Nevertheless in this case we have assumed no errors in the regressors, which is not the case in a practical situation. Next example illustrates this point, and the robustness properties of the algorithm.

Example 4.2: In this other example we focus on the performance of the proposed algorithm with respect to the initial errors in the values of the estimated system parameters θ_0^i. In this case a 6–th. order stable model composed of three SOS has been identified. The coefficients of each of the SOS are given in Table II. The initial values of the model parameters adopted were randomly generated to differ with those above in approximately 15 %. These values are given in Table III.

	SOS1	SOS2	SOS3
b_0	1.4286	1	1
b_1	2.4426	0.39854	-1.3084
b_2	1.2422	1.0125	0.79505
a_0	1	1	1
a_1	1.2682	0.41029	-0.82137
a_2	0.48484	0.50966	0.57812

TABLE II

COEFFICIENTS OF THE SOS REPRESENTED AS TRANSFER FUNCTIONS.

	SOS1	SOS2	SOS3
b_0	1.3857	1.1167	1.0917
b_1	2.0916	0.4562	-1.2241
b_2	1.0648	0.8643	0.8377
a_0	1	1	1
a_1	1.2374	0.3667	-1.0155
a_2	0.4394	0.4455	0.5446

TABLE III

INITIAL COEFFICIENTS OF THE IDENTIFICATION ALGORITHM REPRESENTED AS TRANSFER FUNCTIONS.

The input to the system is a train of pulses and the model and system output are illustrated in figure 4. Note that the algorithm behaves well, taking in account the highly demanding input and the relatively high uncertainty in the initial setting of the parameters. Ulterior simulations have shown that the performance diminishes as the uncertainty or the number of SOS in the cascade grow.

Although we have used the Switching σ modification algorithm, similar results were obtained using in the simulations other robust identification procedures such as the Projection algorithm ([2]). This has also been illustrated in the previous example. This fact shows that the structure of the proposed algorithm: *propagation scheme - robust identification - Nehari projection* is central and the robust identification algorithm applied is of secondary importance.

Example 4.3: Here, a practical example taken from measurements in an acoustic tube are presented. The tube is 4 meters long and has a reference and error microphones located at both extremes. The input signal is produced by an industrial fan and has been measured by the reference microphone located next to it. The output signal has been measured by the error microphone at the other end of the tube. The identification scheme is based in the Projection algorithm ([2]) and the initial coefficients of all SOS sections have been computed from an off–line identification of the complete transfer function based on a parametric–nonparametric technique ([19]). This is a convenient practical

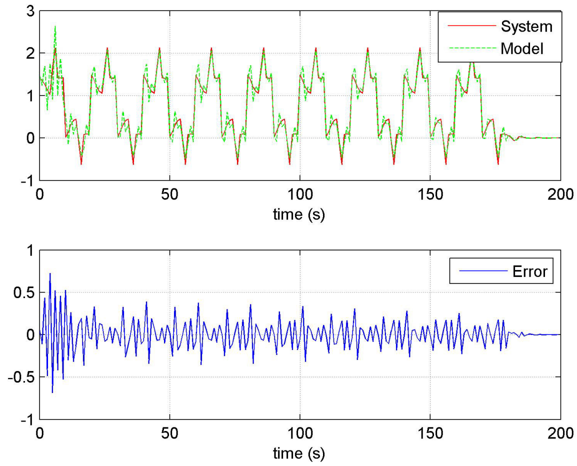

Fig. 4. 6–th. order model approximation using the Switching σ modification algorithm.

approach so that the algorithm is initiated from a close enough neighborhood of the actual parameters. The off-line identification procedure can be anyone which can produce a sufficiently good model of the experimental data, taking advantage of the fact that it does not need to be implemented in real time. The results are presented in figures 5 and 6 and produce a good fit of the experimental data, both in frequency and time.

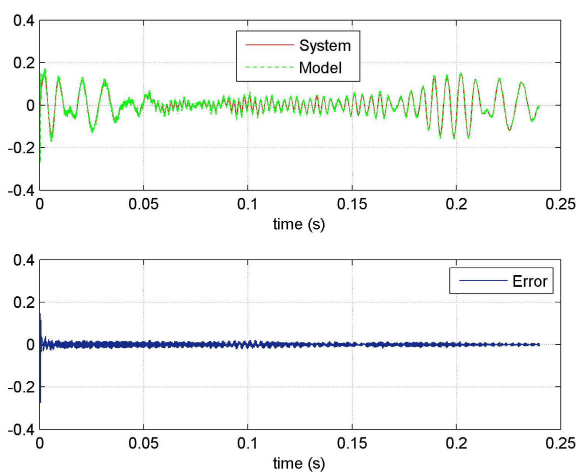

Fig. 5. 12th. order experimental data approximation - time response.

V. Conclusions

We have presented a robust adaptive identification algorithm based on a SOS-cascade realization, with forward and backward propagation of the input and output regresors. Stability of the estimation of each SOS (and as a consequence of the whole cascade) is obtained via a Nehari projection. Simulated and experimental examples illustrate

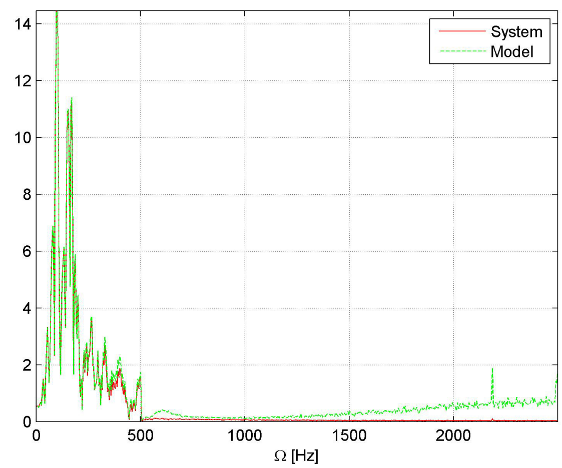

Fig. 6. 12th. order experimental data approximation - frequency response.

the performance of this algorithm. Further research needs to be done to guarantee practical bounds on the regressor initial errors, and hence on the robustness of the procedure.

VI. Acknowledgments

This work is part of the Complementary Agreement between the Advanced Control Systems (SAC) group at UPC (Spain) and the Dept. of Physics and Mathematics and the Control and Systems Center at ITBA (Argentina) signed September 2006. The second author received the support for this work of the Institució Catalana de Recerca i Estudis Avançats (ICREA) and the Spanish CICYT Ref. DPI2005-04722. The first and third authors are partially supported by FONCyT Project 31255 (Argentina).

References

[1] G. Goodwin and K. Sin, *Adaptive Filtering, Prediction and Control*. New Jersey: Prentice Hall, 1984.

[2] G. Tao, *Adaptive Control, Design and Analysis*. New Jersey: John Wiley and Sons, 2003.

[3] S. Kuo and D. Morgan, *Active Noise Control Systems: Algorithms and DSP Implementations*. New York: John Wiley and Sons, 1995.

[4] K. Narenda, Ed., *Adaptive and Learning Systems; Theory and Applications*. New York: Plenum Press, 1986.

[5] P. A. Ioannou and J. Sun, *Robust Adaptive Control*. Prentice Hall, Inc, 1996.

[6] R. Ortega and Y. Tang, "Robustness of Adapative Controllers – a Survey," *Automatica*, vol. 25, no. 5, pp. 651–677, 1989.

[7] P. Kootsookos, R. Bitmead, and M. Green, "The Nehari shuffle: FIR(q) filter design with guaranteed error bounds," *IEEE Transactions on Signal Processing*, vol. 40, pp. 1876–83, 1992.

[8] Y. Yamamoto, B. Anderson, and M. Nagahara, "Optimal FIR approximation for discrete-time IIR filters," in *Proceedings 4th. Asian Control Conference*, ser. FA3, no. 1, Singapore, September 2002, pp. 2008–13.

[9] B. Rao, "Adaptive IIR filtering using cascade structures," in *Proceedings 27th. Asilomar Conference on Signal, Systems, and Computers*, vol. 1, 1993, pp. 194–198.

[10] J. A. G. A. Williamson and M. Nayeri, "Structural issues in cascade-form adaptive iir filters," in *Proceedings International Conference on Acoustics, Speech, and Signal Processing, ICASSP-95*, vol. 2, 1995, pp. 1436–1439.

[11] P. Lueg, *Process of silencing sound oscillations*. U.S. Patent 043,416, 1934.

[12] P. Nelson and S. Elliot, *Active Control of Sound*. Academic Press, 1992.

[13] J. Freudenberg and D. Looze, "Right half plane poles and zeros and design tradeoffs in feedback systems," *IEEE Transactions on Automatic Control*, vol. 30, no. 6, pp. 555–565, 1985.

[14] M. Serón, J. Braslavsky, and G. Goodwin, *Fundamental Limitations in Filtering and Control*. Springer, 1997.

[15] J. Hong and D. Bernstein, "Bode integral constraints, colocation, and spillover in active noise and vibration control," *IEEE Transactions on Control Systems Technology*, vol. 6, no. 1, 1998.

[16] J. S. Freudenberg, C. V. Hollot, R. H. Middleton, and V. Toochinda, "Fundamental design limitations of the general control configuration," *IEEE Transactions on Automatic Control*, vol. 48, no. 8, 2003.

[17] M. A. Cugueró, B. Morcego, and R. S. Sánchez-Peña, *Identification and Control: The Gap between Theory and Practice*. Springer-Verlag, 2007, ch. Identification and Control Structure Design in Active (Acoustic) Noise Control, pp. 199–241.

[18] J. Ball, I. Gohberg, and L. Rodman, *Interpolation of Rational Matrix Functions,Operator Theory: Advances and Applications*. Birkhäuser, 1990, vol. 45.

[19] P. A. Parrilo, R. S. Sánchez Peña, and M. Sznaier, "A parametric extension of mixed time/frequency robust identification," *IEEE Transactions on Automatic Control*, vol. 44, no. 2, pp. 364–369, 1999.