An inference engine based on fuzzy logic for uncertain and imprecise expert reasoning

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Abstract

This paper addresses the development and computational implementation of an inference engine based on a full fuzzy logic, excluding only imprecise quantifiers, for handling uncertainty and imprecision in rule-based expert systems. The logical model exploits some connectives of Lukasiewicz's infinite multi-valued logic and is mainly founded on the work of L.A. Zadeh and J.F. Baldwin. As it is oriented to expert systems, the inference engine was developed to be as knowledge domain independent as possible, while having satisfactory computational efficiency. This is achieved firstly by using the same linguistic term set in every universe of discourse. Thus, it is possible to add a dictionary to the knowledge base, which translates the usual linguistic values of the domain to those of the term set. Secondly, the logical operations of negation and conjunction and the modus ponens rule of inference are implemented exclusively in the truth space.

The approach provides, firstly, a realistic and unambiguous solution to the combination of evidence problem and, secondly, offers two alternative versions of implementation. The full version uses the algorithms of the operations involved. In a more efficient version, which places a small constraint on the use of linguistic modifiers and is confined to knowledge bases whose inference chains are no longer than three links, the above algorithms are replaced by pre-computed tables. A

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1. Introduction

Data that are present in real-world problems can be precise and imprecise, certain and uncertain. Moreover, most knowledge is heuristic and, hence, is usually imprecise and uncertain. Apart from the knowledge being imprecise and uncertain, as a rule, it is either impossible or very expensive to obtain more reliable information. Therefore, the only alternative is to use the information as it is. However, in highly specialised problems, the human expert looks for a solution under these conditions, and, therefore, an expert system (ES) should be equally capable of doing so if it is to emulate the expert.

It is clear from the above just how important it is to have inference engines based on extensions of

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bi-valued logic that permit not only truth grades, as in multi-valued logics, but also the representation, measurement and processing of uncertain and imprecise information. If there is no capability for handling uncertain information, a precise truth value must be selected, just as precise values must be adopted for imprecise information when the capability to measure and process this kind of information is not available. This latter case poses a bigger problem, because if the computational implementation of the system does not have the capability to handle partial pattern matching, it is possible that not all the implicit information will be made explicit for processing. It is conceivable in both cases that the best decision, that is, the best hypothesis from a set of hypotheses in the particular case of ESs, will not be made, leading to lower reliability. Furthermore, while handling uncertainty, which is exclusively connected to the truth value of propositions, involves only one linguistic term set of a single universe -the truth space-, the treatment of imprecision, which depends on the proposition itself, involves an almost infinite number of different universes with different term sets, corresponding to the universes of discourse of the facts involved. This would lead to a huge amount of linguistic values for assignation and, consequently, to a very complicated and inefficient computational implementation.

It is patent that fuzzy logic has gained scientific maturity at the theoretical level. From the technological point of view, however, this success is, except for a very few, shining contributions, less clear-cut. Apart from considering the scientific issues, this paper addresses the validation of ideas that close a major knowledge gap, namely, the development of an effective and computationally efficient inference engine based on fuzzy logic for uncertain and imprecise expert reasoning.

The *solution proposed* in this paper presents the first *development and implementation* of an *inference engine* oriented to *rule-based ESs*, based on a *full fuzzy logic* model [5,26], excluding only imprecise quantifiers [29], for handling both *uncertainty and imprecision*, where a full fuzzy logic means a logic that includes:

(1) Truth values as *fuzzy subsets* of the truth space rather than *precise numbers*, *weights* or *degrees of certainty*. As mentioned under point 5.1.1.1, the term set must contain five values: "absolutely_false" ("false" in bi-valued logic), "false", "undecided", "true" and "absolutely_true" ("true" in bi-valued logic). The others must be intercalated between the above. Therefore, *fuzzy numbers* cannot be used either.
(2) *Fuzzy pattern matching*, managed by means of *Zadeh's compositional modus ponens* [26].
(3) *Imprecise quantifiers*.
(4) Some acceptable solution to the problem of *combination of evidence*.

It follows from the above that uncertainty is confined to possibilistic uncertainty as far as a model based on fuzzy logic is concerned.

The inference engine offers a *realistic and unambiguous solution* to the *combination of evidence problem*, an *efficient implementation*, which makes the computational implementation of a system of this sort *feasible*, and, finally, is largely *independent of knowledge domains*, without sacrificing the possibility of a *user-friendly interface*, which enables the system to be queried using the usual linguistic values of the domain.

The fuzzy *logic model* is mainly based on papers by Zadeh [5,22–26,28] and Baldwin [1–4], addressing the treatment of uncertainty and imprecision, oriented to rule-based ES applications. It is feasible to adopt Lukasiewicz's implication in order to achieve a close *generalisation of bi-valued logic* [13,15] for the stated problem. The reason is that the logic operations are carried out exclusively in the truth space, which does not exacerbate inefficiency as happens when this implication is used for processing in the universes of discourse. For the *combination of evidence* problem, a solution is proposed that is based on a fuzzy logic extension of an approach by Dubois and Prade [11] for the possibilistic model, which always leads to three precisely defined cases: consistency, partial inconsistency and total inconsistency. It was not foreseen that the inference engine should handle exceptions in the rules.

In order to achieve *computational efficiency*, the generalised engine operations corresponding to negation, conjunction and modus ponens are carried out exclusively in the truth space, along with the operations of truth functional modification (TFM) [5] and inverse truth functional modification (ITFM) [1]. The possible irreversibility of these two operations interfacing between the truth space and the universes of discourse [9], if any, always leads to the most conservative (the least restricted) conclusion. The combination, on the one hand, of an adequate term set of nine linguistic truth values with parabolic membership functions, generated by adding the value *undecided* to the other eight values obtained by applying the linguistic modifiers [6,21] "fairly", "very" and "absolutely" to the values "true" and "false", and, on the other, the use of only one term set of linguistic values with properly overlapped trapezoidal functions in all universes of discourse and the same linguistic modifiers as for truth values, simplifies the operations interfacing between the truth space and the universes of discourse and assures that ITFM operation results are in accordance with what should be expected. As modus ponens and ITFM can output non-parabolic functions in some cases, such engine operations are followed by a parabolic approximation operation to ensure that every truth-value always has a parabolic function.

All universes of discourse have the same term set of five linguistic values ("tiny", "small", "medium", "large" and "huge") of properly overlapped trapezoidal membership functions to assure *independence of knowledge domains*. The use of the same three linguistic modifiers as in the truth space increases the spectrum of standard values to twenty.

A dictionary has been foreseen in every knowledge base to produce a *user-friendly interface*. It is, therefore, possible to interact with the system using the usual linguistic values of the universe of discourse to which the fact in question belongs. The dictionary then translates these to the linguistic values of the term set.

Finally, the conjunction of the solutions in the truth space and in the universes of discourse provides for *two versions of implementation*. The full version uses the algorithms of the operations involved. In a more efficient version, the above algorithms are replaced by pre-computed tables, constructed by applying linguistic approximation to each value obtained after applying the algorithms of the non-closed operations. This version, which places a small constraint on the use of linguistic modifiers and is confined to knowledge bases whose inference chains are no longer than three links, has been incorporated into an automatic theorem prover, fully implemented in Prolog. On the other hand, the main program of the full version has been implemented in Prolog and the calculation algorithms in C.

2. State of the art

At present, most fuzzy logic applications, generally referred to as fuzzy systems, are essentially oriented to control (fuzzy control), and only a few address ESs. This is mainly due to the fact that only in control systems has it been possible to produce efficient implementations. The reason for this is that control systems always operate with certain information and precise data, where it is clearly possible to use term sets of similar linguistic values and of standard functions in all universes of discourse and, furthermore, Mamdani's implication (min-conjunction) [13,18] can be adopted. On the other hand, ESs must operate with certain and uncertain information, precise and imprecise data. This means that there are usually many universes of discourse with different term sets. Finally, some implication function other than Mamdani's (for instance, Lukasiewicz's or Kleene-Dienes's implication) should be adopted in order to assure a logic model that comes as close as possible to a generalisation of bi-valued logic [15] for the stated problem. All of this makes the operation of the modus ponens rule of inference considerably more complex and inefficient in inference engines operating in universes of discourse, especially as the antecedents of the rules are more complex [16]. Truth space operation does not have this problem and normally provides much higher efficiency [1]. However, this alternative is more or less ruled out by the presence of ITFM operations, since if membership functions in the truth space and in the universes of discourse are not properly selected, the results output by those operations are very likely not to be totally in accordance with what should be expected.

All the above problems of ES-oriented fuzzy systems have meant that *implementations capable of handling uncertainty and imprecision are not yet based on a full fuzzy logic*, as defined in Section 1, which could be considered to be a close generalisation of bi-valued logic, as essential in this type of applications. Some examples of systems of this kind are: *PROTIS* [20], *DIABETO* [7], *Z-II* [17] and, more recently, *RESYFU* [14].

The PROTIS System uses fuzzy rules whose "if" part represents imprecise concepts by means of a fuzzy subset, chosen by the expert from four standard subsets, and whose "Then" and "Else" parts make use of *precise numbers* provided by the expert, such as independent *weights* of evocation and rejection. Fuzzy pattern matching is possible, and is based on the use of *possibility and necessity measures*. The DIABETO *System* is based on the model developed by Dubois and Prade [10], which combines possibilistic logic for handling uncertainty with Zadeh's compositional modus ponens for handling imprecision. The Z-II System uses *degrees of certainty* with two alternatives: *precise numbers or fuzzy numbers*, allowing representation of imprecise facts in data and in rule antecedents and consequents, and making inferences by means of a *modified* compositional rule of inference. The RESYFU System (Reasoning System with Fuzzy Uncertainty) works with knowledge represented in the form of "If–Then" rules accompanied by *uncertainty degrees* expressed by *fuzzy numbers*. It should be noted that some other systems, like MILORD [12], are out of the scope of this comparison because they handle merely uncertainty and not impreciseness.

It is clear then that none of the above systems includes a full fuzzy logic as defined in Section 1.

Finally, as an example of very recent trends, which attest to the search for other alternatives to the numerical approach of approximate reasoning, due to its implementation problems, the *SAR System* (symbolic approximate reasoning) [8] should be mentioned. This system enlarges on previous work by Schwartz concerning the *symbolic treatment of approximate reasoning* [19]. In the referenced paper, the authors of SAR discuss the huge effort that has gone into properly implementing aspects of approximate reasoning theory for use in *expert reasoning systems* and conclude by saying: "*Yet none have so far come near to implementing it in its entirety*".

3. Problem statement

This paper seeks to develop and implement an inference engine that solves the following general problem of *uncertain and imprecise expert* reasoning based on *fuzzy logic*, limited only by the exclusion of imprecise quantifiers:

To find the value of $A_3(X_3)$ from:

(1) $(A_1(X_1)$ is $P_1^1 \wedge \neg A_2(X_2)$ is $P_2^1 \Rightarrow A_3(X_3)$ is $P_3^1)$ is τ_{I1},
(2) $(A_4(X_4)$ is $P_4^1 \Rightarrow A_3(X_3)$ is $P_3^2)$ is τ_{I2},
(3) $(A_1(X_1)$ is $P_1^2)$ is τ_1^2,
(4) $(A_2(X_2)$ is $P_2^2)$ is τ_2^2,
(5) $(A_4(X_4)$ is $P_4^2)$ is τ_4^2,

$/\tau_{I1}, \tau_{I2}, \tau_1^2, \tau_2^2, \tau_4^1 \subseteq V \equiv [0,1] =$truth space: fuzzy sets
A_1, A_2, A_3, A_4: attributes X_1, X_2, X_3, X_4: objects
$P_1^1, P_1^2 \subset U_1; P_2^1, P_2^2 \subset U_2; P_3^1, P_3^2, P_3^3 \subset U_3; P_4^1, P_4^2 \subset U_4$: fuzzy sets satisfying all the following *conditions*, designed to attain an *implementable general-purpose inference engine*:

- a close generalisation of bi-valued logic for the stated problem,
- good computational efficiency,
- possibility of a realistic and unambiguous combination of evidence,
- large independence of knowledge domains,
- provision for a user-friendly interface.

Imprecise quantifiers have not been included for the system in question on two grounds. First, they can be accounted for in most cases, albeit less expressively, by adopting suitable linguistic truth-values. Second, their inclusion would have seriously complicated and detracted from the fulfilment of the above conditions, especially as far as computational efficiency and combination of evidence are concerned. With a view to preserving expressiveness, however, we have not yet abandoned the idea of imprecise quantifiers and a new version of the inference engine with such quantifiers is planned, once a detailed analysis of the logic model and its respective computational processing has been completed.

The solution to this problem, divided into three stages (logic model definition, inference engine development and implementation) is shown in Sections 4–6, respectively.

4. Definition of the logic model

Of the five conditions imposed on inference engine development and implementation in Section 3, the logic model must define what should be done in order to satisfy the first three conditions. With respect to a close *generalisation of bi-valued logic*, the model is a generalisation of Lukasiewicz's multi-valued logic. Concerning *computational efficiency*, the operations of negation and conjunction and the modus ponens rule of inference are applied in the truth space. With regard to the *combination of evidence*, a new fuzzy logic approach is presented, based on the concept of degree of consistency, which makes it possible to arrive at three unambiguous cases of combination, which are compatible with reality.

Defining a logic model for the purpose described in this paper involves defining the six operations to be performed by the engine, that is, negation, conjunction, modus ponens, as it operates in the truth space, TFM and ITFM, and combination of evidence. The expressions of the first five operations are shown below under points 4.1 and 4.2, based on the papers by Zadeh [5,26] and Baldwin [1,3], already mentioned in Section 1. Finally, the expressions for the combination of evidence are shown under point 4.3.

4.1. Negation, conjunction and modus ponens

- *Negation*

$$\frac{\neg(p \text{ is } \tau)}{p \text{is } \tau_{\mathrm{N}} = \mathrm{Nv}(\tau)}$$

$$/\mu_{\tau_{\mathrm{N}}}(v_{\mathrm{N}}) = \mu_{\tau}(1 - v_{\mathrm{N}}). \tag{4.1}$$

- *Conjunction*

$$\frac{(p_1 \text{ is } \tau_1) \wedge (p_2 \text{ is } \tau_2)}{(p_1 \wedge p_2) \text{ is } \tau_{12} = \mathrm{Cv}(\tau_1, \tau_2)}$$

$$/\mu_{\mathrm{t}_{12}}(v_{12}) = \mathrm{Max}\left[\mathrm{Min}\left[\mu_{\mathrm{t}_1}(v_{12}), \underset{(v_2 \in [v_{12}, 1])}{\mathrm{Max}} \mu_{\mathrm{t}_2}(v_2)\right], \mathrm{Min}\left[\mu_{\mathrm{t}_2}(v_{12}), \underset{(v_1 \in [v_{12}, 1])}{\mathrm{Max}} \mu_{\mathrm{t}_1}(v_1)\right]\right]. \tag{4.2}$$

- *Modus ponens*

$$\frac{\begin{array}{l}(p_1 \Rightarrow p_2) \text{ is } \tau_{\mathrm{I}} \\ p_1 \text{ is } \tau_1\end{array}}{p_2 \text{ is } \tau_2 = \tau_1(V_1) \circ I_{\tau_{\mathrm{I}}}(V_1 \times V_2) = \mathrm{PPv}(\tau_1, \tau_{\mathrm{I}})}$$

$$/\mu_{\tau_2}(v_2) = \underset{(v_1)}{\mathrm{Max}}[\mathrm{Min}[\mu_{\tau_1}(v_1), \mu_{\tau_{\mathrm{I}}}(\mathrm{Min}[1, 1 - v_1 + v_2])]], \tag{4.3}$$

where for all the above expressions p_i is the proposition, μ_{τ_i} the membership function, V the truth space, Nv the generalised negation operation in the truth space, Cv the generalised conjunction operation in the truth space and PPv the generalised modus ponens rule of inference in the truth space.

4.2. Interface operations

- *TFM*

$$\frac{(A(X) \text{ is } P) \text{ is } \tau}{A(X) \text{ is } R = \mathrm{TFM}(P, \tau)}$$

$$/\mu_{\mathrm{R}}(u) = \mu_{\tau}(\mu_{\mathrm{P}}(u)). \tag{4.4}$$

- *ITFM*

$$\frac{\begin{array}{l}A_1(X_1) \text{ is } P_1^1 \\ A_1(X_1) \text{ is } P_1^2\end{array}}{V(A_1(X_1) \text{ is } P_1^1 / A_1(X_1) \text{ is } P_1^2)}$$

$$= \tau = \mathrm{ITFM}(P_1^1, P_1^2)$$

$$/\mu_{\tau}(v) = \underset{(u_1)}{\mathrm{Max}}\ \mu_{P_1^2}(u_1) / \mu_{P_1^1}(u_1) = v, \tag{4.5}$$

where $\mathrm{TFM}(P, \tau)$: TFM operation applied to P and τ, $V(A_1(X_1)$ is $P_1^1/A_1(X_1)$ is $P_1^2)$: truth-value of $A_1(X_1)$ is P_1^1, since $A_1(X_1)$ is P_1^2 is true, $\mathrm{ITFM}(P_1^1, P_1^2)$: ITFM operation applied to P_1^1 and P_1^2.

4.3. Combination of evidence

For the system proposed in this paper, this operation arises when during a query, for instance, there is a set of premises as follows:

(1) $A_1(X_1)$ is $P_1^1 \Rightarrow A_2(X_2)$ is P_2^1,
(2) $A_3(X_3)$ is $P_3^1 \Rightarrow A_2(X_2)$ is P_2^2,

(3) $A_1(X_1)$ is P_1^1,
(4) $A_3(X_3)$ is P_3^1,

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whence:

(i) $A_2(X_2)$ is P_2^1,
(ii) $A_2(X_2)$ is P_2^2.

Assuming, firstly, that there is *no interaction* between Propositions (1) and (2) [25]:

$$A_2(X_2) \text{ is } P_2^3 = P_2^1 \cap P_2^2$$
$$/\mu_{P_2^3}(u_2) = \mathrm{Min}[\mu_{P_2^1}(u_2), \mu_{P_2^2}(u_2)]. \tag{4.6}$$

If there is any dependency between the relations, the knowledge of the relation between $A_2(X_2)$ and other variables, like $A_1(X_1)$ and $A_3(X_3)$, should be expressed by rules of the type $A_1(X_1)$ is $P_1^1 \wedge A_3(X_3)$ is $P_3^1 \Rightarrow A_2(X_2)$ is P_2^1 and not by rules that involve $A_1(X_1)$ and $A_3(X_3)$ separately.

As the result of expression (4.6) is always a value that is at most equal to or, generally, more restricted than the values to be combined, its application does not always lead to consistent results.

The combination of evidence problem in fuzzy logic is still an *open problem*. This paper proposes a solution to this problem, implemented in an inference engine. This solution is based on a fuzzy logic extension of an approach by Dubois and Prade for the possibilistic model, based on expression (4.6). This expression represents the *conjunctive combination mode*, which is the starting point for deciding which kind of combination is to be made, according to the presumed *reliability* of the available knowledge, expressed by the results derived from the calculation of the intersection of the subsets involved, where a distinction should be made between the following *three cases*:

(a) the intersection is a *normalised* subset,
(b) the intersection is a *subnormalised but not empty* subset,
(c) the intersection is *empty*.

With respect to the above three cases, it can be inferred that:

(α) Result (a) normally indicates an important overlapping of the membership functions of the intersected sets, and hence the presence of reliable sources of information and of a *consistent* data and knowledge base. Therefore, the application of expression (4.6) leads to a satisfactory result, since it arrives at a more restricted value.

(β) Result (b) shows that, even though the intersection is not empty, there is no one value that is fully possible for both sources, indicating, therefore, a conflict between the sources and a *partial inconsistency* of the base and data. The application of expression (4.6) in this case always leads to a more restricted value, a result that is incompatible with the low reliability of the information. It is reasonable under these circumstances to follow Dubois and Prade's advice of adopting the *disjunctive combination mode*:

$$A_2(X_2) \text{ is } P_2^3 = P_2^1 \cup P_2^2$$
$$/\mu_{P_2^3}(u_2) = \mathrm{Max}[\mu_{P_2^1}(u_2), \mu_{P_2^2}(u_2)] \tag{4.7}$$

since this expression leads to a less restricted value, which is more compatible with the conditions in question and with reality. Semantically, this result simply expresses indecision between the values represented by P_2^1 and by P_2^2, respectively.

(γ) Result (c) indicates a very severe conflict between the sources and a totally inconsistent base and data. Therefore, nothing can be asserted about the value of the linguistic variable involved.

To summarise this point:

$$\begin{array}{l} A_2(X_2) \text{ is } P_2^1, \\ A_2(X_2) \text{ is } P_2^2, \\ \hline A_2(X_2) \text{ is } \mathrm{CEd}(P_2^1, P_2^2), \end{array}$$

where CEd, combination of evidence operation in the universe of discourse, involves, firstly, carrying out the operation indicated by expression (4.6) and then, if necessary, applying the operations under (β) or, possibly, (γ). Any tools that incorporate this approach should give the user the alternative to change or confirm the data causing the conflict, when either of the latter two cases are detected, before delivering the result of the combination.

4.4. Solution to the problem

By transforming the problem in Section 3 for solution in the truth space, it is firstly possible to write:

(1) $(A_1(X_1)$ is $P_1^1 \wedge \neg A_2(X_2)$ is $P_2^1 \Rightarrow A_3(X_3)$ is $P_3^1)$ is τ_{I1},
(2) $(A_4(X_4)$ is $P_4^1 \Rightarrow A_3(X_3)$ is $P_3^2)$ is τ_{I2},
(3) $(A_1(X_1)$ is $P_1^1)$ is $\mathrm{ITFM}(P_1^1, \mathrm{TFM}(P_1^2, \tau_1))$,
(4) $(A_2(X_2)$ is $P_2^1)$ is $\mathrm{ITFM}(P_2^1, \mathrm{TFM}(P_2^2, \tau_2^2))$,
(5) $(A_4(X_4)$ is $P_4^1)$ is $\mathrm{ITFM}(P_4^1, \mathrm{TFM}(P_4^2, \tau_4^2))$,

which should be rewritten as shown below:

(1) $(p_1 \wedge \neg p_2 \Rightarrow A_3(X_3)$ is $P_3^1)$ is τ_{I1},
(2) $(p_4 \Rightarrow A_3(X_3)$ is $P_3^2)$ is τ_{I2},
(3) p_1 is $\mathrm{ITFM}(P_1^1, \mathrm{TFM}(P_1^2, \tau_1^2))$,
(4) p_2 is $\mathrm{ITFM}(P_2^1, \mathrm{TFM}(P_2^2, \tau_2^2))$,
(5) p_4 is $\mathrm{ITFM}(P_4^1, \mathrm{TFM}(P_4^2, \tau_4^2))$.

It follows from the above points 4.1–4.3 that the solution *at the logic model level* is derived by composing operations, through steps (6)–(10), as follows:

(6) $(A_3(X_3)$ is $P_3^1)$ is $\mathrm{PPv}(\mathrm{Cv}(\mathrm{ITFM}(P_1^1, \mathrm{TFM}(P_1^2, \tau_1^2)), \mathrm{Nv}(\mathrm{ITFM}(P_2^1, \mathrm{TFM}(P_2^2, \tau_2^2)))), \tau_{I1})$,
(7) $(A_3(X_3)$ is $P_3^2)$ is $\mathrm{PPv}(\mathrm{ITFM}(P_2^1, \mathrm{TFM}(P_2^2, \tau_2^2)), \tau_{I2})$,
(8) $A_3(X_3)$ is $\mathrm{TFM}(P_3^1, \mathrm{PPv}(\mathrm{Cv}(\mathrm{ITFM}(P_1^1, \mathrm{TFM}(P_1^2, \tau_1^2)), \mathrm{Nv}(\mathrm{ITFM}(P_2^1, \mathrm{TFM}(P_2^2, \tau_2^2)))), \tau_{I1}))$,
(9) $A_3(X_3)$ is $\mathrm{TFM}(P_3^2, \mathrm{PPv}(\mathrm{ITFM}(P_2^1, \mathrm{TFM}(P_2^2, \tau_2^2)), \tau_{I2}))$,
(10) $A_3(X_3)$ is $\mathrm{CEd}(\mathrm{TFM}(P_3^1, \mathrm{PPv}(\mathrm{Cv}(\mathrm{ITFM}(P_1^1, \mathrm{TFM}(P_1^2, \tau_1)), \mathrm{Nv}(\mathrm{ITFM}(P_2^1, \mathrm{TFM}(P_2^2, \tau_2^2)))), \tau_{I1}))\mathrm{TFM}, (P_3^2, \mathrm{PPv}(\mathrm{ITFM}(P_2^1, \mathrm{TFM}(P_2^2, \tau_2^2)), \tau_{I2})))$.

5. Development of the inference engine

While the logic model is directly responsible for the first three conditions of the problem, the development of the engine must satisfy the last two conditions imposed in Section 3. With respect to the *independence of knowledge domains*, the proposal is to use the same term set in all universes of discourse. With regard to *a user-friendly interface*, provision is made for a dictionary in each knowledge base for translating the usual values of the knowledge domain to those of the term set. Furthermore, the conjunction of the solutions in the truth space and in the universes of discourse means that computational efficiency can be further improved, a realistic and unambiguous solution for the combination of evidence problem can be found, the results of ITFM operations are in accordance with what should be expected and, as an extra facility, two alternative implementations are offered: the full version, which calculates the algorithms of the operations, and a limited version, which uses pre-computed tables.

Having defined the logic model, the development of a fuzzy logic-based inference engine is fundamentally concerned with solving two problems: the problem of defining the linguistic term sets in the truth space and in the universes of discourse, discussed under point 5.1, and the problem of linguistic approximation, discussed under 5.2. Finally, the solution to the problem, according to the definitions given under the above points, is shown under 5.3.

5.1. Linguistic term sets

Defining the linguistic term sets involves, firstly, defining the quality and quantity of the linguistic terms on the basis of the analysis and discussion of the qualitative and quantitative conditions that should and must be satisfied and, secondly, defining the most suitable membership functions for ensuring that the above conditions are also met for the values modified by engine operations.

5.1.1. Quality and quantity of terms

5.1.1.1. In the truth space. First, five linguistic truth-values naturally emerge from the definition of a fuzzy logic model Baldwin [1], as shown in Fig. 1. These are "absolutely_false" (af), "false" (f), "undecided" (u), "true" (t) and "absolutely_true" (at) [1,9]. Hence, they will have to belong to the set to be adopted. This implies that the problem of defining the term set is circumscribed to the best choice of the values to be intercalated between the above five values.

The following conditions should be imposed on the linguistic terms to be adopted and their respective fuzzy sets in order to solve the *qualitative problem*:

(1) That they are normalised subsets.
(2) That they have an easy semantic interpretation.
(3) That if a truth value emerges out of the adopted set as a result of non-closed logic operations, the determination of its greater or lesser falsehood

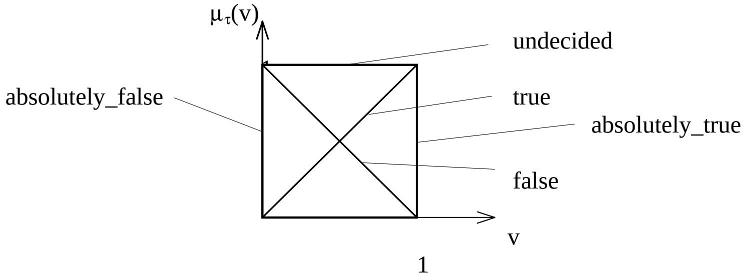

Fig. 1. Linguistic truth values.

or truth with respect to any other value, whether or not it belongs to the adopted set, is neither imprecise nor too complex.

(4) That they produce a progressive and ordered TFM, from the value "false" up to the value "true" of bi-valued logic, and also meet the conditions adopted in the universes of discourse.
(5) That the respective membership functions are easily represented and implemented computationally.
(6) That they make up a symmetric term set, with respect to the value "undecided".

Concerning the *quantitative problem*, experience has shown that human beings cannot discern many more than nine levels. Therefore, so as to make as finer decomposition as possible and satisfy the first part of condition (4), the number of terms should be odd and symmetric with respect to the value *undecided*, and the first integer that meets these conditions is *nine*. So, four values, two and two, respectively on either side of the values *false* and *true*, must be added to the five values mentioned at the beginning of this point, and these can be defined as *fairly-false* and *very-false*, on either side of *false* and *fairly-true* and *very-true* on either side of *true*.

5.1.1.2. In the universes of discourse. It is clear from Section 1 that some new approach should be developed in order to enable the implementation of system applications such as the ones addressed by this paper, where we are looking for as much domain independence as possible, as well as good computational efficiency. An interesting approach to the universes of discourse problem is, on the one hand, to adopt the same linguistic term set for all universes and, on the other hand, to incorporate a dictionary in each knowledge base, adapted to the domain in question in order to provide a user-friendly interface and be able to interact with the system using the usual linguistic values of the universe of discourse to which the fact involved belongs. These are translated by the dictionary to the values of the adopted term set. If this is the chosen solution, the use of linguistic modifiers should be added in order to expand the spectrum of permitted values.

Once a decision has been made to adopt a single term set, it still remains to define which and how many linguistic terms should be selected. As for the truth-values, the solution to the *qualitative problem* emerges from the imposition of the *conditions* to be satisfied by the term set, as discussed below.

(1) That they are normalised subsets.
(2) That they have an easy semantic interpretation.
(3) That the determination of the relative value with respect to the other values for all linguistic values derived from TFM operations is neither imprecise nor too complex.
(4) That they produce an ITFM that satisfies the following conditions in order to be in accordance with what should be expected:

(a) $$\mathrm{ITFM}(m_i\_\delta_\mathrm{h}, \mathrm{TFM}(m_j\_\delta_\mathrm{h}, \tau_k)) = \tau \in T, \quad /\tau_k \in T, \tag{5.1}$$

(b) $$\mathrm{ITFM}(m_i\_\delta_\mathrm{h}, \mathrm{TFM}(m_j\_\mathrm{Ad}(\delta_\mathrm{h}), \tau_k)) = \tau \in F, \quad /\tau_k \in T, \tag{5.2}$$

(c) $\mathrm{ITFM}(m_i\_\delta_{\mathrm{h}}, \mathrm{TFM}(m_j\_\mathrm{Ad}^n(\delta_{\mathrm{h}}), \tau_k) = \mathrm{af},$

$$/\tau_k \in T, \ \ n \geqslant 2, \tag{5.3}$$

(d) $\mathrm{ITFM}(m_i\_\delta_{\mathrm{h}}, \mathrm{TFM}(m_j\_\delta_{\mathrm{h}}, \tau_k)) = \tau \in F,$

$$/\tau_k \in F, \tag{5.4}$$

(e) $\mathrm{ITFM}(m_i\_\delta_{\mathrm{h}}, \mathrm{TFM}(m_j\_\mathrm{Ad}(\delta_{\mathrm{h}}), \tau_k)) = u,$

$$/\tau_k \in F, \tag{5.5}$$

(f) $\mathrm{ITFM}(m_i\_\delta_{\mathrm{h}}, \mathrm{TFM}(m_j\_\mathrm{Ad}^n(\delta_{\mathrm{h}}), \tau_k)) = u,$

$$/\tau_k \in F, \ \ n \geqslant 2, \tag{5.6}$$

where $\delta_{\mathrm{h}} \in D =$ set of linguistic values in universes of discourse, $m_i, m_j \in \mathrm{M} =$ set of linguistic modifiers, $\mathrm{Ad}(\delta_{\mathrm{h}})$ is the adjacent value to δ_{h}, $\mathrm{Ad}^n(\delta_{\mathrm{h}})$ is the nth adjacent value to δ_{h}, T the subset of true truth values(truer than "u"), F the subset of false truth values (more false than "u").

The meaning of the above condition can be clarified by means of an example of condition (a): ITFM (very_large, TFM(fairly_large, very_true)) should be a true truth-value in order to be in accordance with what should be expected.

(5) That the respective membership functions are easily represented and implemented computationally.

In addition to the above *conditions*, the following conditions are also *recommended*:

(6) That they make up a symmetric term set, with respect to an intermediate value.
(7) That the intermediate linguistic terms have symmetric functions with respect to the unitary central value and that the extreme linguistic terms have asymmetric functions: the lowest term having a unitary value at zero, and the highest term having a unitary value at the right end.
(8) That the direct linguistic modifiers in the universes of discourse be the same as in the truth space: "fairly", "very" and "absolutely", subject to $m_i\_\delta =$ "δ is m_i_true", in order to simplify and better close all the transformations between the truth space and the universes of discourse.
(9) That the inverse linguistic modifiers "not_fairly", "not", "not_very" and "not_absolutely" be excluded for reasons of semantic and computational simplicity.

Concerning the *quantitative* problem, it can be inferred from (6) that the number must be odd. The first integer that satisfies this condition is three, which is evidently very low. On the other hand, experience has shown that seven is excessive for these purposes and so five values were finally adopted: "medium", as the central value, "tiny" and "huge", as the lowest and highest values, respectively, "small" between the lowest and middle values, and "large" between the middle and highest values. Finally, the total number of standard values is twenty due to the presence of the three linguistic modifiers that emerge from the satisfaction of condition (8).

5.1.2. Membership functions

The solutions adopted in the truth space and in the universes of discourse, shown under points 5.1.2.1 and 5.1.2.2, respectively, emerged from the analysis of the conditions and recommendations discussed under points 5.1.1.1 and 5.1.1.2 and from the study of the behaviour of operations, analysed under point 5.1.2.3, especially non-closed operations, as these are the operations that could cause problems if the output functions do not belong to the family of input functions.

5.1.2.1. In the truth space. Pursuant to the above, a set of nine values τ was adopted as the linguistic term set: absolutely-false (af), very-false (vf), false (f), fairly-false (ff), undecided (u), fairly-true (ft), true (t), very-true (vt) and absolutely-true (at), of parabolic functions: $\mu_\tau(v) = v^n$ for the true values and $\mu_\tau(v) = (1-v)^n$ for the false values. In order to achieve a progressive TFM, the exponent 3 was adopted for the values "vf" and "vt", and $\frac{1}{3}$ for "ff" and "ft".

5.1.2.2. In the universes of discourse. The only set adopted is composed, as shown on the left-hand side of Fig. 2, by five basic terms δ (*tiny*, *small*, *medium*, *large and huge*) of trapezoidal functions that meet the overlapping condition — $(\forall\delta)(\mu_\delta(u) = 1 \Rightarrow \mu_{\mathrm{Ad}(\delta)}(u) = 0)$ —, required to satisfy the ITFM operation conditions (5.1)–(5.6).

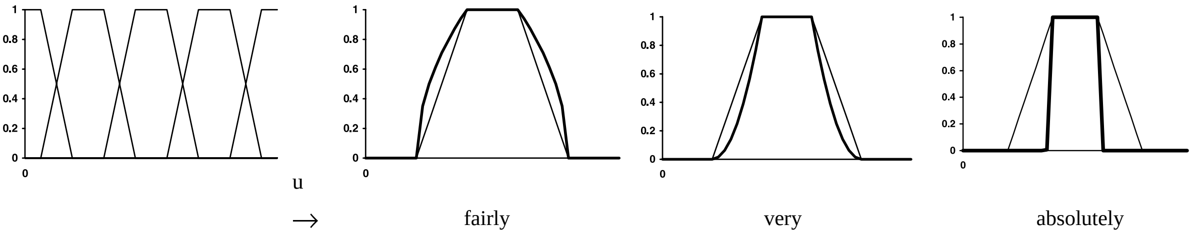

Fig. 2. Combination of parabolic and trapezoidal functions.

Three linguistic modifiers m (*fairly*, *very and absolutely*), identical to the truth space modifiers, which satisfy $m\_\delta = \delta$ is m_true, raise the number of standard values to twenty. The right-hand side of Fig. 2 shows each resultant value of this modification, obtained by applying TFM operations.

It is important to note that the combination of parabolic functions in the truth space and trapezoidal functions with the overlapping condition in universes of discourse assures that the modified set has the same supporting set as the baseline set in TFM operations, as shown in Fig. 2. If polygonal rather than parabolic functions had been adopted, the supporting set would not be maintained and the other values would be totally invaded (which means that the overlapping condition would not be satisfied for the modified values), as it can be seen from the application of expression (4.4) and from Fig. 3, which shows the functions $\mu_R(u)$ output after applying TFM operations to a linguistic value of a trapezoidal membership function $\mu_P(u)$ and the truth value *fairly-true*, when parabolic and polygonal functions are adopted for the truth values.

Inverse linguistic modifiers were not included for reasons of semantic and computational simplicity.

5.1.2.3. Operations behaviour. The approach to this study was based on analysing, firstly, whether or not the operation is a closed operation and, if not, whether or not its output function belongs to the adopted family of functions.

(a) *Closed operations*: Negation, conjunction and combination of evidence belong to this class of operations, as explained below.

Concerning negation, the application of expression (4.1) to a function which belongs to the term set must always be another function of the term set, because of condition (6) of point 5.1.1.1.

With respect to conjunction, the adopted truth values belong to the class of truth value restrictions proposed by Baldwin [1] and, hence, the result of applying expression (4.2) to two or more values is always the most false value [3], which result is also a standard function.

As regards combination of evidence, it follows from point 4.3 that there are only three precisely defined cases of combination, as per cases (α), (β) and (γ) of the logic model. The consistency case (α) arises when only similar values are processed. The case of partial inconsistency (β) arises when there is at least one value close to similar values. The case of total inconsistency (γ) arises when there is at least one pair of neither similar nor close values. It follows from the above and point 5.1.2.2 that the outputs of this operation are: the most restricted value in case (α) and the disjunction of close values in case (β). It is impossible to output anything in case (γ).

(b) *Non-closed operations*: All other engine operations, corresponding to modus ponens, ITFM and TFM fall into this class. Modus ponens delivers polygonal rather than parabolic functions when outputting truth values between the value undecided and the value true, as follows from the application of expression (4.3) [1,9]. ITFM meets the conditions under point 5.1.1.2, but it does not deliver parabolic functions when outputting false values (conditions (5.2) and (5.4)), as follows from applying expression (4.5) to the cases in question. TFM delivers non-standard functions. However, very importantly, it maintains the same supporting set as the baseline value for modifications like TFM(m_δ, τ), where τ belongs to the subset of false values, as follows from the application of expression (4.4) to such cases.

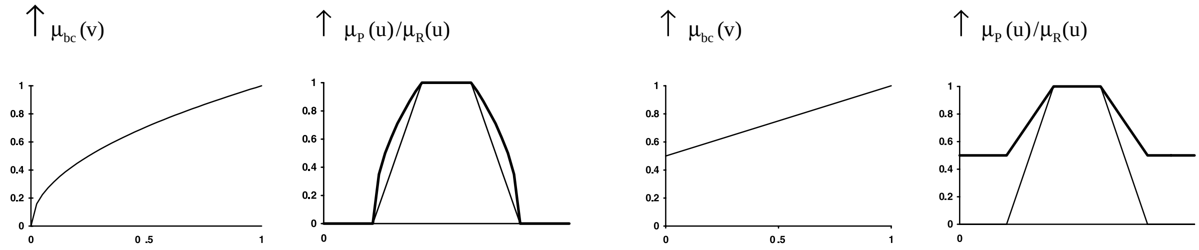

Fig. 3. Output $\mu_R(u)$ functions.

(c) *Parabolic approximation*: As a result of the above, every time that a modus ponens rule of inference or ITFM operation generates a non-parabolic function $g(v)$, this function has to be replaced by the parabolic function that determines the same area as $g(v)$ in the real interval $[0, 1]$, and thus all truth values will have parabolic functions. This parabolic approximation means that, for the modified values, the behaviour of the operations will be unchanged and the conditions will be met.

5.2. Linguistic approximation

Any linguistic approximation process should answer two questions: *when* and *how* it should be carried out it. There are two alternative answers to the *when* question. One, referred to as the *first alternative*, involves approximating after each final conclusion, while the other, the *second alternative*, involves approximating after each non-closed operation. The first alternative outputs more reliable results, although it is less efficient. The second alternative, on the other hand, has the disadvantage that the approximation error increases through rule chaining. However, it has an important advantage of computational efficiency, since only standard values are processed, which means that pre-computed tables can be implemented instead of algorithms to calculate the operations. Having compared the use of both alternatives, we found that, except for a very few cases, the results do not differ, even with three inference levels.

The answer to the *how* question is approximation procedures. The *closeness criterion* in both the truth space and the universes of discourse is minor area difference in the interval concerned.

5.3. Solution to the problem

The development of this inference engine is concluded by presenting the problem and its solution according to the above considerations. Point 5.3.1 addresses the first alternative and point 5.3.2, the second alternative. For didactic reasons, the knowledge base is assumed to have no inconsistent cases as discussed under points 4.3 and 5.1.2.3 (a). Under these conditions, the problem addressed in Section 3 is set out according to the standard values and linguistic modifiers.

5.3.1. The first alternative

To find the value of $A_3(X_3)$, from:

(1) $(A_1(X_1)$ is $m_k^1\text{-}\delta_j^1 \wedge \neg A_2(X_2)$ is $m_k^2\text{-}\delta_j^2 \Rightarrow A_3(X_3)$ is $m_k^3\text{-}\delta_j^3)$ is τ_{iv}^1,
(2) $(A_4(X_4)$ is $m_k^4\text{-}\delta_j^4 \Rightarrow A_3(X_3)$ is $m_k^5\text{-}\delta_j^5)$ is τ_{iv}^2,
(3) $(A_1(X_1)$ is $m_k^6\text{-}\delta_j^6)$ *is* τ_i^1,
(4) $(A_2(X_2)$ is $m_k^7\text{-}\delta_j^7)$ is τ_i^2,
(5) $(A_4(X_4)$ is $m_k^8\text{-}\delta_j^8)$ is τ_i^4,

where $\tau_i^{n1} \in S$ is the truth value set, $\tau_{\text{iv}}^{n2} \in T$ the subset of true values, $\delta_j^{n3} \in D$ the linguistic term set in universes of discourse and $m_k^{n4} \in M$ the linguistic modifiers set, where subscripts i, j, k indicate any value belonging to the respective set, and superscripts $n1, n2, n3$ and $n4$ simply indicate the order in which the value concerned appears.

By following steps (6)–(9) discussed under point 4.4, step (10) gives the *solution* to the above problem:

(10) $A_3(X_3)$ is CEd(Ad(TFM($m_k^3\text{-}\delta_j^3$, Ap(PPv(Cv (Ap(ITFM($m_k^1\text{-}\delta_j^1$, TFM($m_k^6\text{-}\delta_j^6$, τ_i^1)))), Nv(Ap (ITFM($m_k^2\text{-}\delta_j^2$, TFM($m_k^7\text{-}\delta_j^7$, τ_i^2))))), τ_{iv}^1))), Ad

$(\mathrm{TFM}(m_k^5\text{-}\delta_j^5,\ \mathrm{Ap}(\mathrm{PPv}(\mathrm{Ap}(\mathrm{ITFM}(m_k^4\text{-}\delta_j^4,\ \mathrm{TFM}(m_k^8\text{-}\delta_j^8,\ \tau_i^4)))),\tau_{\mathrm{iv}}^2))))),$

where "Ad" is a linguistic approximation operation in the universe of discourse and "Ap" a parabolic approximation operation.

5.3.2. *The second alternative*

In order to limit the size of the TFM and ITFM arrays, the problem has to be constrained in this case so as to ensure that each TFM operation is always applied to linguistic values without any modifier in the universes of discourse and to only standard truth values. For this purpose, linguistic modifiers are only permitted in facts when the truth value is "true" and in the antecedent of the rules. It should be noted that this limitation is not too restrictive in practice, as it simply rules out doubly imprecise cases like "($A(X)$ is $m\text{-}\delta$) is τ".

According to the above, the problem is set out according to the second alternative as follows:

Find the value $m_k^9\text{-}\delta_j^9$ of $A_3(X_3)$ from:

(1) $(A_1(X_1)$ is $m_k^1\text{-}\delta_j^1 \wedge \neg A_2(X_2)$ is $m_k^2\text{-}\delta_j^2 \Rightarrow A_3(X_3)$ is $\delta_j^3)$ is τ_{iv}^1,
(2) $(A_4(X_4)$ is $m_k^3\text{-}\delta_j^4 \Rightarrow A_3(X_3)$ es $\delta_j^5)$ is τ_{iv}^2,
(3) $(A_1(X_1)$ is $\delta_j^6)$ is τ_i^1,
(4) $(A_2(X_2)$ is $\delta_j^7)$ is τ_i^2,
(5) $(A_4(X_4)$ is $\delta_j^8)$ is τ_i^4.

It is important to point out that although TFM is, generally, a non-closed operation, the constraints imposed on this version make it a closed operation and, hence, it is not necessary to use linguistic approximation. Thus, it follows that the pre-computed tables must be constructed for the operations negation, conjunction, TFM, ITFM with linguistic approximation, combination of evidence and the modus ponens rule of inference with linguistic approximation, which are inferred from the respective expressions under points 4.1–4.3.

From all the above considerations and following the procedure under points 4.4 and 5.3.1, the solution to the above problem is obtained in step 10, as follows:

(10) $(A_3(X_3)$ is $\mathrm{CEd}(\mathrm{TFM}(m_k^3\text{-}\delta_j^3,\mathrm{Av}(\mathrm{PPv}(\mathrm{Cv}(\mathrm{Av}(\mathrm{ITFM}(m_k^1\text{-}\delta_j^1,\mathrm{TFM}(m_k^6\text{-}\delta_j^6,\tau_i^1))),\mathrm{Nv}(\mathrm{Av}(\mathrm{ITFM}(m_k^2\text{-}\delta_j^2,\mathrm{TFM}(m_k^7\text{-}\delta_j^7,\tau_i^2)))))),\tau_{\mathrm{iv}}^1)),\mathrm{TFM}(m_k^5\text{-}\delta_j^5,\mathrm{Av}(\mathrm{PPv}(\mathrm{Av}(\mathrm{ITFM}(m_k^4\text{-}\delta_j^4,\mathrm{TFM}(m_k^8\text{-}\delta_j^8,\tau_i^4)))),\tau_{\mathrm{iv}}^2))),$

where "Av" represents a linguistic approximation operation in the truth space.

6. Implementation

A version of Prolog was selected for the computational implementation of the engine, which is both an interpreter and compiler and also has a suitable C interface. Being an interpreter and compiler, it provided for an efficient implementation of the *second alternative*, entirely developed in Prolog by means of a *meta-interpreter* dedicated to this application. Having a suitable C interface, it also provided for an efficient implementation of the *first alternative*, which was based on the same meta-interpreter, developed in C, from which the algorithms of the operations involved are called.

As far as knowledge representation is concerned, predicates named "is1" and "is2" were used to handle uncertainty and imprecision, respectively. Therefore, a general fact like "($A(X)$ is $m\text{-}\delta$) is τ" is represented by the clause:

$\mathrm{is1}(\mathrm{is2}(a(x), m\text{-}\delta), \tau)$

and a general rule like "$(A_1(X_1)$ is $m_1\text{-}\delta_1 \Rightarrow A_2(X_2)$ is $m_2\text{-}\delta_2)$ is τ" is represented by the clause:

$\mathrm{is1}(\mathrm{is2}(a2(x2), m2\text{-}\delta 2), \tau) :- \mathrm{is2}(a1(x1), m1\text{-}\delta 1).$

Finally, a fact with a crisp proposition like "p is τ" is represented by the clause "is1(p, τ)".

7. Experimentation

Each of the two versions of the inference engine were incorporated into backward-chaining automatic theorem-prover programs. A series of executions of the system based on the second alternative, entirely developed in Prolog, are discussed below. They show the different cases of consistency, partial inconsistency and total inconsistency that can arise in respect of the cases (α), (β) and (γ) described under points 4.3

and 5.1.2.3 (a). The examples were put together by means of a generic base, which is shown below.

is1(is2($a1(x1)$,huge), vt) :- not(is2($a2(x2)$, tiny)),
  is2($a3(x3)$, f_small).
is1(is2($a1(x1)$, huge), at) :- is2($a4(x4)$, v_large).
is1(is2($a1(x1)$, large), t) :- $p5$, is2($a6(x6)$, small).
is1(is2($a1(x1)$, small), at) :- not($p7$).
is1($p8$, at) :- $p9$.
is1(is2($a10(x10)$, large), t) :- is2($a1(x1)$, huge).
is1(is2($a10(x10)$, large), vt) :- not($p11$),
  is2($a1(x1)$, f_large), $p8$.
is1(is2($a10(x10)$, medium), at) :- $p12$.
is1(is2($a10(x10)$, small), ft) :- is2($a1(x1)$, v_small),
  not(is2($a13(x13)$, v_tiny)).
is1(is2($a10(x10)$, small), at) :- $p14$.

The examples shown below are ordered from consistency, through partial inconsistency, to total inconsistency. Example (a) is a case of consistency: $a10(x10)$ is f_large and $a10(a10)$ is large. Example (b) is a case of partial inconsistency, where the deduction is continued after a partial inconsistency in intermediate conclusions ($a1(x1)$ is v_huge and $a1(x1)$ is large), is equivalent to adopting the union of the sets as combination. Example (c) is a case of partial inconsistency in the final conclusions: $a10(x10)$ is f_large and $a10(x10)$ is medium. Examples (d) and (e) are cases of total inconsistency. Firstly, example (d) illustrates total inconsistency in intermediate conclusions ($a1(x1)$ is v_huge and $a1(x1)$ is abs_small) and, secondly, example (e) total inconsistency in final conclusions ($a10(x10)$ is f_large and $a10(x10)$ is abs_small). The reasoning and, hence, all the conclusions are fully invalid in the last two examples. Therefore, it is possible to assert nothing about the value of $a10(x10)$.

In all cases, the information entered by the user is written in italics.

(a) **Consistency**

(1) Write the CONJECTURE:
  ?*is*2($a10(x10)$, *Value*).

(2) Write the known FACTS:
  - *is*2($a2(x2)$, *large*.
  - *is*2($a3(x3)$, *v_small*).
  - *is*2($a4(x4)$, abs*_large*).
  - *stop.*

(3) Intermediate conclusion:
  $a1(x1)$ is v_huge

(4) Partial final conclusion:
  $a10(x10)$ is f_large

(5) Intermediate conclusion:
  $a1(x1)$ is abs_huge

(6) Partial final conclusion:
  $a10(x10)$ is large

(7) FINAL CONCLUSION:

  $a10(x10)$ is large
yes

(b) **Partial inconsistency by intermediate conclusion**

(1) Write the CONJECTURE:
  ?*is*2($a10(x10)$, *Value*).

(2) Write the known FACTS:
  - is2($a2(x2)$, *large*).
  - *is*2($a3(x3)$, *v_small*).
  - *is*1($p11$, *af*).
  - *is*1($p5$, *at*).
  - *is*2($a6(x6)$, abs*_small*).
  - *is*1($p9$, *t*).
  - *stop.*

(3) Intermediate conclusion:
  $a1(x1)$ is v_huge

(4) Partial final conclusion:
  $a10(x10)$ is f_large

(5) Intermediate conclusion:
  $a1(x1)$ is large

(6) Intermediate conclusion:
  p8 is vt

(7) Partial final conclusion:
  $a10(x10)$ is large

(8) FINAL CONCLUSION:
$a10(x10)$ is large

yes

(c) **Partial inconsistency by final conclusion**

(1) Write the CONJECTURE:
$?is2(a10(x10), Value)$.

(2) Write the known FACTS:
- $is2(a2(x2), large)$.
- $is2(a3(x3), v\_small)$.
- $is1(p12, t)$.
- *stop.*

(3) Intermediate conclusion:
$a1(x1)$ is v_huge

(4) Partial final conclusion:
$a10(x10)$ is f_large

(5) Partial final conclusion:
$a10(x10)$ is medium

(6) Knowledge base is PARTIALLY INCONSISTENT.
FINAL CONCLUSION
is undecided between:
$a10(x10)$ is:
EITHER f_large
OR medium

yes

(d) **Total inconsistency by intermediate conclusion**

(1) Write the CONJECTURE:
$?is2(a10(x10), Value)$.

(2) Write the known FACTS:
- $is2(a2(x2), large)$.
- $is2(a3(x3), v\_small)$.
- $is1(p7, af)$.
- *stop.*

(3) Intermediate conclusion:
$a1(x1)$ is v_huge

(4) Partial final conclusion
$a10(x10)$ is f_large

(5) Intermediate conclusion:
$a1(x1)$ is abs_small

(6) Above CONCLUSIONS
are NOT VALID.
Knowledge base is
FULLY INCONSISTENT.
$a1(x1)$ is:
EITHER v_huge
OR abs_small
Then it is possible to assert
nothing about
$a10(x10)$

no

(e) **Total inconsistency by final conclusion**

(1) Write the CONJECTURE
$?is2(a10(x10), Value)$.

(2) Write the known FACTS:
- $is2(a2(x2), large)$.
- $is2(a3(x3), v\_small)$.
- $is1(p14, at)$.
- *stop.*

(3) Intermediate conclusion:
$a1(x1)$ is v_huge

(4) Partial final conclusion:
Then it is possible to assert
nothing about $a10(x10)$
$a10(x10)$ is f_large

(5) Partial final conclusion:
$a10(x10)$ is abs_small

(6) Above CONCLUSIONS are
NOT VALID.
Knowledge base is
FULLY INCONSISTENT.
$a10(x10)$ is:
EITHER f_large
OR abs_small

no

8. Conclusions

This paper proposes a solution to the development of fuzzy logic-based inference engines oriented to ES, which, for the first time, effectively and efficiently solves the problem of uncertain and imprecise reasoning based on a full fuzzy logic devoted to ESs, whose sole limitation is that it excludes imprecise quantifiers. Therefore, it solves the problems of computational inefficiency, knowledge domain dependency and combination of evidence that have been an obstacle to successful application of a full fuzzy logic to ESs.

The solution proposed by this paper to the above problem is based on:

1. The logic model is a generalisation of Lukasiewicz's infinite multi-valued logic, confined to the goal of being a model devoted to rule-based ESs, and operates in the truth space. For the combination of evidence, the model extends Dubois and Prade's approach for the possibilistic model to fuzzy logic. This model always leads to realistic results and to only three precisely defined cases of consistency, partial inconsistency and total inconsistency.
2. A term set of nine linguistic values with parabolic functions is used in the truth space, and the same term set of five linguistic values with trapezoidal functions (which meet a suitable overlapping condition to assure that ITFM operations output results that are in accordance with what should be expected), plus the same three linguistic modifiers as in the truth space are used in each universe of discourse. Truth values with parabolic functions make sure that TFM operations output values with the same supporting set as the baseline set, which means that the above values also meet the overlapping condition. Hence, ITFM also outputs results that are in accordance with what should be expected, using modified values. As the analysis of the behaviour of the engine operations showed that modus ponens and ITFM were non-closed operations and could output non-parabolic functions, it was decided to perform a parabolic approximation operation after carrying out each of these operations. TFM can also output non-standard functions, but maintains the same supporting set, which obviates any problem.
3. The use of the same linguistic term set with modifiers in all universes of discourse makes the inference engine operation easier and largely independent from the knowledge domains.
4. The feasibility of including a dictionary in knowledge bases provides for the use of the usual terminology of each domain and also a user-friendly interface.

The combination of the solutions attained in the truth space and universes of discourse, together with the selected logic model, provides for an efficient computational implementation, even with large inference chains and rules with complex antecedents, and outputs results that are in accordance with what should be expected in ITFM operations. Moreover, it allows the implementation of a very efficient version that employs pre-computed tables instead of the operation algorithms, involving a small constraint on the use of linguistic modifiers and a loss of reliability when inference chains are longer than three links.

Future research and development areas will be directed, on the one hand, at adding imprecise quantifiers, after conducting a detailed analysis of the logic model and its respective computational processing. Apart from completing the model, this will make it possible to increase expressiveness for certain propositions. On the other hand, concerning the extension of the system developed, there are the two obvious extensions for an inference engine such as is presented here: firstly, its incorporation into an ES development and consultation tool and, secondly, the incorporation of a natural language interface to the above tool. The availability of such a tool will evidently enable performance of the necessary field trials to arrive at a realistic and reliable evaluation of this type of systems.

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